Eur. Phys. J. B **41**, 185–195 (2004)
DOI: 10.1140/epjb/e2004-00308-5

THE EUROPEAN
PHYSICAL JOURNAL B

On the Meissner effect in a superconductor with 4-fermion attraction

P. Tarasewicz[a]

Faculty of Pharmacy, The Ludwik Rydygier Medical University in Bydgoszcz, ul. Jagiellońska 13, 85-067 Bydgoszcz, Poland

Received 10 March 2004 / Received in final form 14 June 2004
Published online 12 October 2004 –

Abstract. A presence of a Meissner-Ochsenfeld effect in a gas of spin 1/2 fermions with an interaction $V_4 = -|\Lambda|^{-1} \sum_{\mathbf{k},\mathbf{k}'} g_{\mathbf{k},\mathbf{k}'} b^*_{\mathbf{k}} b^*_{-\mathbf{k}} b_{\mathbf{k}'} b_{-\mathbf{k}'}$, where $|\Lambda|$ is a volume of a region Λ in real space which is taken by the system and $b^*_{\mathbf{k}} = a_{\mathbf{k}+} a_{\mathbf{k}-}$ with $a_{\mathbf{k}\sigma}$, $a^*_{\mathbf{k}'\sigma'}$ satisfying Fermi anticommutation relations, is investigated. The effect proves to be weaker than in BCS by a factor 3/4 at $T = 0$, implying a greater penetration depth λ of external magnetic field. V_4 is nonzero only within a thin layer of 1-fermion energies around the chemical potential μ.

PACS. 74.20.-z Theories and models of superconducting state – 74.20.Fg BCS theory and its development

1 Introduction

A proposal to extend the Bardeen-Cooper-Schrieffer theory of superconductivity by incorporating into the BCS Hamiltonian H_{BCS} a 4-fermion BCS-type attraction between Cooper pairs

$$V_4 = -|\Lambda|^{-1} \sum_{\mathbf{k},\mathbf{k}'} g_{\mathbf{k}\mathbf{k}'} b^*_{\mathbf{k}} b^*_{-\mathbf{k}} b_{-\mathbf{k}'} b_{\mathbf{k}'}, \tag{1.1}$$

where

$$b_{\mathbf{k}} = a_{\mathbf{k}+} a_{\mathbf{k}-}, \qquad g_{\mathbf{k}\mathbf{k}'} = g\chi(\mathbf{k})\chi(\mathbf{k}'), \qquad g > \mathbf{0},$$

(with $\chi(\mathbf{k})$ denoting the characteristic function of the set $\{\mathbf{k} : \mu - \delta \le \varepsilon_{\mathbf{k}} \le \mu + \delta\}$ for $\varepsilon_k = \frac{\hbar^2 k^2}{2m}$ and fixed δ) and potential of Spałek, Wójcik [2,3]

$$W = \sum_{\mathbf{k}} \gamma_{\mathbf{k}} n_{\mathbf{k}+} n_{\mathbf{k}-},$$

where

$$n_{\mathbf{k}\sigma} = a^*_{\mathbf{k}\sigma} a_{\mathbf{k}\sigma}, \qquad \sigma = \pm$$

was made in 1996 [1] by Maćkowiak and the author. Spałek and Wójcik introduced interaction W in order to obtain so called statistical spin liquid in which $\gamma_k = U > 0$ and $U \to \infty$. In the case of statistical spin liquid interaction W leads to the exclusion of double occupied configurations with the same $\mathbf{k}$ in reciprocal space.

In [4,5] Czerwonko suggested to complete BCS Hamiltonian with interaction W in which $U \to \infty$, getting a particular case of statistical spin liquid. In the model presented here γ_k can in general be assumed to be an arbitrary real function of $\mathbf{k}$ but so far we have restricted ourselves to the case in which the interaction W itself is nonzero only within a thin layer around the chemical potential by using $\gamma_k = \gamma\chi(\mathbf{k})$. Moreover, γ was assumed to be negative or zero as it is in this paper. The function $\chi(\mathbf{k})$ allows us to impose the restriction on a range of the interactions in the reciprocal space.

Appearance of the interaction V_4 in the Hamiltonian of Czerwonko can be justified by some suggestions and discoveries. Namely, in 1993 Schneider and Keller [6] measured the different features of some cuprates and Chevrel-phases superconductors, especially concentrating on a relation between the critical temperature and zero temperature condensate density. They noticed that the experimental data for e.g. $YBa_2Cu_3O_{6.602}$ point to the behaviour of a dilute Bose gas. As a result they suggested Bose condensation of weakly interacting fermion pairs as a mechanism of transition from normal to superconducting state. Moreover, one needs to mention a recent discovery of Bunkov et al. [7] which points to presence of fermion quadruples in ^{3}He. Their work was devoted to the problem of influence of spatial disorder on the order parameter in superfluid ^{3}He. The authors quoting Volovik [8] suggested that unusual spectra of ^{3}He in aerogel could be explained by process in which impurities tend to destroy the anisotropic correlations of the order parameter, while correlations of higher symmetry can survive (e.g. four-particle correlations). It is worth pointing to another recent paper of Schneider at al. [9] in which a discovery of half-$h/2e$ magnetic flux in SQUIDs fabricated of bicrystalline $YBa_2Cu_3O_{7-\delta}$ films is reported. As it is known this

[a] e-mail: `tarasek1@amb.bydgoszcz.pl`

situation corresponds to a presence of fermion quadruples in the system and leads to taking the interplay between Cooper pairs and quadruples into consideration. In model presented here at sufficiently low temperatures uncorrelated pairs (as a result of potential W with $\gamma < 0$), Cooper pairs and fermion quadruples are present in the system. The interaction V_4 can be seen as either an attraction between Cooper pairs or fermion pairs caused by W. Such interaction can be mediated by different hypothetic boson fields, e.g. phonons. A microscopic derivation as well as a nature of this interaction is under investigation at present.

In subsequent papers [10–14] some aspects of thermodynamics of the Hamiltonian $H_{BCS} + V_4 + W$ were studied. In particular, it was demonstrated in [12] that $H_{BCS} + V_4 + W$ is a mean-field Hamiltonian which allows exact solution and that the resulting thermodynamics bears some similarities to the thermal behaviour of high-temperature superconductors (HTSC), e.g. the presence of a pseudogap in the excitation spectrum and two order parameters. The existence of two order parameters agrees with the proposal given by Müller in [15] in order to explain experimental data on the symmetry of the order parameter in HTSC. The thermodynamics of $H_{BCS}+V_4+W$ with vanishing BCS-interaction

$$V_{BCS} = -|\Lambda|^{-1} \sum_{\mathbf{k},\mathbf{k}'} G_{\mathbf{k}\mathbf{k}'} a^*_{\mathbf{k}+} a^*_{-\mathbf{k}-} a_{-\mathbf{k}'-} a_{\mathbf{k}'+} = 0,$$

where $G_{\mathbf{k}\mathbf{k}'}$ has the same form as $g_{\mathbf{k}\mathbf{k}'}$, was studied in [13,14]. Analogies with HTSC behaviour were also found, e.g. weak character of transition to superconducting state which can be 1st or 2nd order, convexity of the critical field in the vicinity of T_c, linearity of specific heat in a wide range temperatures below T_c.

The present paper deals with the question of a Meissner-Ochsenfeld (MO) effect in the Fermi gas with the 4-fermion interaction V_4:

$$H_4 = \sum_{\mathbf{k}\sigma} \xi_k a^*_{\mathbf{k}\sigma} a_{\mathbf{k}\sigma} + V_4,$$

where $\xi_k = \varepsilon_k - \mu$ and μ is chemical potential. In this case the excitation spectrum differs from the BCS one, although one can find operators α such that

$$\alpha |G\rangle_{\mathbf{k}} = 0,$$

where $|G\rangle_{\mathbf{k}}$ is the ground state vector for momentum $\mathbf{k}$. Its form is presented in Appendix A as the vector number 15. Using new notation $a_{\mathbf{k}1} = a_{\mathbf{k}+}$, $a_{\mathbf{k}2} = a_{\mathbf{k}-}$, $a_{\mathbf{k}3} = a_{-\mathbf{k}+}$, $a_{\mathbf{k}4} = a_{-\mathbf{k}-}$ operators α can be written in the following form

$$\begin{aligned}
\alpha_{\mathbf{k}1} &= u_{\mathbf{k}} a_{\mathbf{k}1} - v_{\mathbf{k}} a^*_{\mathbf{k}2} a^*_{\mathbf{k}3} a^*_{\mathbf{k}4},\\
\alpha_{\mathbf{k}2} &= u_{\mathbf{k}} a_{\mathbf{k}2} + v_{\mathbf{k}} a^*_{\mathbf{k}3} a^*_{\mathbf{k}4} a^*_{\mathbf{k}1}\\
\alpha_{\mathbf{k}3} &= u_{\mathbf{k}} a_{\mathbf{k}3} - v_{\mathbf{k}} a^*_{\mathbf{k}1} a^*_{\mathbf{k}2} a^*_{\mathbf{k}4},\\
\alpha_{\mathbf{k}4} &= u_{\mathbf{k}} a_{\mathbf{k}4} + v_{\mathbf{k}} a^*_{\mathbf{k}1} a^*_{\mathbf{k}2} a^*_{\mathbf{k}3},
\end{aligned}$$

where $u^2_{\mathbf{k}} = \frac{1}{2}(1 + \frac{\xi_k}{E_{\mathbf{k}}})$, $v^2_{\mathbf{k}} = \frac{1}{2}(1 - \frac{\xi_k}{E_{\mathbf{k}}})$, $E_{\mathbf{k}} = \sqrt{\xi^2_{\mathbf{k}} + \Delta^2}$ and Δ is the order parameter which is defined in Section 2.

These operators fulfil the following anticommutation rules

$$\begin{aligned}
\{\alpha_{\mathbf{k}i}, \alpha_{\mathbf{k}'j}\} &= \alpha_{\mathbf{k}i}\alpha_{\mathbf{k}'j} + \alpha_{\mathbf{k}'j}\alpha_{\mathbf{k}i} = 0 \quad \text{for} \quad i,j = 1,2,3,4\\
\{\alpha_{\mathbf{k}i}, \alpha^*_{\mathbf{k}j}\} &= v^2_{\mathbf{k}} a^*_{\mathbf{k}j} a_{\mathbf{k}i} \left(1 - \sum_{l\neq i,j} n_{\mathbf{k}l}\right) \quad \text{for} \quad i \neq j\\
\{\alpha_{\mathbf{k}i}, \alpha^*_{\mathbf{k}i}\} &= 1 + v^2_{\mathbf{k}} \left(\sum_{i\neq j<l\neq i} n_{\mathbf{k}j} n_{\mathbf{k}l} - \sum_{j\neq i} n_{\mathbf{k}j}\right)
\end{aligned}$$

where $n_{\mathbf{k}i} = a^*_{\mathbf{k}i} a_{\mathbf{k}i}$. Thus the excitations represented by the operators $\alpha^*_{\mathbf{k}i}$ are neither fermions nor bosons. The normalized $\mathbf{k}$-excited states are therefore represented by the vectors

$$|E_{\mathbf{k}i}\rangle := \alpha^*_{\mathbf{k}i} |G\rangle_{\mathbf{k}} = a^*_{\mathbf{k}i}|0\rangle \tag{1.2}$$

$$|E_{\mathbf{k}ij}\rangle := u^{-1}_{\mathbf{k}} \alpha^*_{\mathbf{k}i}\alpha^*_{\mathbf{k}j} |G\rangle_{\mathbf{k}} = a^*_{\mathbf{k}i} a^*_{\mathbf{k}j}|0\rangle \tag{1.3}$$

$$|E_{\mathbf{k}ijl}\rangle := u^{-2}_{\mathbf{k}} \alpha^*_{\mathbf{k}i}\alpha^*_{\mathbf{k}j}\alpha^*_{\mathbf{k}l} |G\rangle_{\mathbf{k}} = a^*_{\mathbf{k}i} a^*_{\mathbf{k}j} a^*_{\mathbf{k}l}|0\rangle \tag{1.4}$$

$$|E_{\mathbf{k}1234}\rangle := u^{-2}_{\mathbf{k}} \alpha^*_{\mathbf{k}1}\alpha^*_{\mathbf{k}2}\alpha^*_{\mathbf{k}3}\alpha^*_{\mathbf{k}4} |G\rangle_{\mathbf{k}} = (u_{\mathbf{k}} b^*_{\mathbf{k}} b^*_{-\mathbf{k}} - v_{\mathbf{k}})|0\rangle. \tag{1.5}$$

Using eigenstructure from Table 1 in Appendix A one can calculate the excitation energies from the ground state $|G\rangle_{\mathbf{k}}$. They are equal

$$\langle E_{\mathbf{k}i}|H|E_{\mathbf{k}i}\rangle - E_G = 2E_{\mathbf{k}} - \xi_k \tag{1.6}$$

$$\langle E_{\mathbf{k}ij}|H|E_{\mathbf{k}ij}\rangle - E_G = 2E_{\mathbf{k}} \tag{1.7}$$

$$\langle E_{\mathbf{k}ijl}|H|E_{\mathbf{k}ijl}\rangle - E_G = 2E_{\mathbf{k}} + \xi_k \tag{1.8}$$

$$\langle E_{\mathbf{k}1234}|H|E_{\mathbf{k}1234}\rangle - E_G = 4E_{\mathbf{k}}. \tag{1.9}$$

It follows that, unlike in BCS theory, these energies are not simply additive when counted in the $\mathbf{k}$-space spanned by $|G_{\mathbf{k}}\rangle = (u_{\mathbf{k}} + v_{\mathbf{k}} b^*_{\mathbf{k}} b^*_{-\mathbf{k}})|0\rangle$ and the vectors (1.2)–(1.5).

Keeping in mind above facts and using Schafroth's criterion for the MO effect [16,17] it is shown that the effect is present in the system H_4, but is weaker than in the BCS model by a factor 3/4 at $T = 0$.

2 Interaction with external electromagnetic field

The considerations of this section are founded on the method presented in the book by Rickayzen [17]. We shall focus our interest on the Hamiltonian H asymptotically equivalent to H_4 [12], viz.,

$$H = \sum_{\mathbf{k}>0} \left(\xi_{\mathbf{k}} \sum_{\sigma} (n_{\mathbf{k}\sigma} + n_{-\mathbf{k}\sigma}) - 2\Delta_{\mathbf{k}} (B_{\mathbf{k}} + B^*_{\mathbf{k}})\right) + C = \sum_{\mathbf{k}>0} H_{\mathbf{k}} + C,$$

where $B_{\mathbf{k}} = b_{-\mathbf{k}} b_{\mathbf{k}}$, $\Delta_{\mathbf{k}} = |\Lambda|^{-1} \sum_{\mathbf{k}'} g_{\mathbf{k}\mathbf{k}'} \langle B_{\mathbf{k}'}\rangle = \Delta$ and C is a constant. Since H appears in the sequel only in the exponential $\exp(-\beta H)$ of the thermal average, the constant C can be discarded. The equation for Δ (the gap equation) was solved numerically and results were shown in a graphical form in [13].

Our objective is to demonstrate existence of the Meissner-Ochsenfeld effect in the system described by the Hamiltonian $H_f = H + H'$, where H' represents the perturbation due to a weak external electromagnetic field described by the vector potential $\mathbf{A}(\mathbf{r}, t)$. Thus

$$H' = \frac{1}{2m} \int d^3 r \psi^*(\mathbf{r}) \Big[(-i\hbar\nabla - e\mathbf{A}/c)^2 - (-i\hbar\nabla)^2 \Big] \psi(\mathbf{r}).$$

After neglecting the term quadratic in $\mathbf{A}$, one obtains

$$H' = -\frac{e\hbar}{2mc} \sum_{\mathbf{k},\mathbf{k}',\sigma} \mathbf{a}(\mathbf{k}' - \mathbf{k}, t) \cdot (\mathbf{k}' + \mathbf{k}) a^*_{\mathbf{k}'\sigma} a_{\mathbf{k}\sigma}, \quad (2.1)$$

where

$$\mathbf{a}(\mathbf{k}' - \mathbf{k}, t) = 1/|\Lambda| \int d^3 r \mathbf{A}(\mathbf{r}, t) \exp\left[-i(\mathbf{k}' - \mathbf{k}) \cdot \mathbf{r}\right],$$

$|\Lambda|$ denoting the system's volume. In terms of new creation and annihilation operators $a^*_{\mathbf{k}i}$, $a_{\mathbf{k}i}$ (defined as $a_{\mathbf{k}1} = a_{\mathbf{k}+}$, $a_{\mathbf{k}2} = a_{\mathbf{k}-}$, $a_{\mathbf{k}3} = a_{-\mathbf{k}+}$, $a_{\mathbf{k}4} = a_{-\mathbf{k}-}$) H' assumes the form

$$\begin{aligned} H' = -\frac{e\hbar}{2mc} \sum_{\substack{\mathbf{k}>0 \\ \mathbf{k}'>0}} \Big\{ & \mathbf{a}(\mathbf{k}' - \mathbf{k}, t) \cdot (\mathbf{k}' + \mathbf{k})(a^*_{\mathbf{k}'1} a_{\mathbf{k}1} + a^*_{\mathbf{k}'2} a_{\mathbf{k}2}) \\ & + \mathbf{a}(-\mathbf{k}' + \mathbf{k}, t) \cdot (-\mathbf{k}' - \mathbf{k})(a^*_{\mathbf{k}'3} a_{\mathbf{k}3} + a^*_{\mathbf{k}'4} a_{\mathbf{k}4}) \\ & + \mathbf{a}(-\mathbf{k}' - \mathbf{k}, t) \cdot (-\mathbf{k}' + \mathbf{k})(a^*_{\mathbf{k}'3} a_{\mathbf{k}1} + a^*_{\mathbf{k}'4} a_{\mathbf{k}2}) \\ & + \mathbf{a}(\mathbf{k}' + \mathbf{k}, t) \cdot (\mathbf{k}' - \mathbf{k})(a^*_{\mathbf{k}'1} a_{\mathbf{k}3} + a^*_{\mathbf{k}'2} a_{\mathbf{k}4}) \Big\}, \end{aligned}$$

where $\{k : \mathbf{k} > 0\}$ denotes the set of all 1-fermion momenta restricted to a definite half-space of $\boldsymbol{R}^3$.

The system's response to the perturbation H' is given by the thermal expectation value $\mathbf{J}(\mathbf{r}, t)$ of the current density $\hat{\mathbf{J}}(\mathbf{r}, t)$. Weakness of the external field allows to restrict the perturbation expansion for $\mathbf{J}(\mathbf{r}, t)$ to first order

$$\mathbf{J}(\mathbf{r}, t) = \frac{\mathrm{Tr}\, e^{-\beta H} \hat{\mathbf{J}}(\mathbf{r}, t)}{\mathrm{Tr}\, e^{-\beta H}} - \frac{i}{\hbar} \frac{\int_0^t dt' \, \mathrm{Tr}\, e^{-\beta H} \Big[\hat{\mathbf{J}}(\mathbf{r}, t), H'(t') \Big]}{\mathrm{Tr}\, e^{-\beta H}}, \quad (2.2)$$

where $\hat{\mathbf{J}}(\mathbf{r}, t)$ end $H'(t')$ are in the Heisenberg picture. The current density operator is defined as

$$\hat{\mathbf{J}}(\mathbf{r}) = \frac{e}{2m} \Big\{ \psi^*(\mathbf{r})(-i\hbar\nabla - e\mathbf{A}/c)\psi(\mathbf{r}) - \left[(-i\hbar\nabla + e\mathbf{A}/c)\psi^*(\mathbf{r}) \right] \psi(\mathbf{r}) \Big\}$$

which in terms of $a^*_{\mathbf{k}\sigma}$, $a_{\mathbf{k}'\sigma}$ takes the form

$$\begin{aligned} \hat{\mathbf{J}}(\mathbf{r}) &= \frac{e\hbar}{2m|\Lambda|} \sum_{\mathbf{k},\mathbf{k}',\sigma} \exp\left[i(\mathbf{k}' + \mathbf{k}) \cdot \mathbf{r}\right] (\mathbf{k}' + \mathbf{k}) a^*_{\mathbf{k}\sigma} a_{\mathbf{k}'\sigma} \\ &\quad - \frac{e^2}{mc|\Lambda|} \mathbf{A}(\mathbf{r}) \sum_{\mathbf{k}\sigma} a^*_{\mathbf{k}\sigma} a_{\mathbf{k}\sigma} \\ &= \hat{\mathbf{J}}_1 + \hat{\mathbf{J}}_2 = \sum_{\mathbf{q}} \hat{\mathbf{J}}(\mathbf{q}) \exp\left[i\mathbf{q} \cdot \mathbf{r}\right], \end{aligned} \quad (2.3)$$

with $\mathbf{q} = \mathbf{k}' - \mathbf{k}$. Similarly as H', $\hat{\mathbf{J}}(\mathbf{r})$ can be expressed in terms of $a^*_{\mathbf{k}i}$, $a_{\mathbf{k}'i}$, viz.,

$$\begin{aligned} \hat{\mathbf{J}}(\mathbf{r}) = & \\ & \frac{e\hbar}{2mc} \sum_{\substack{\mathbf{k}>0 \\ \mathbf{k}'>0}} \Big\{ \exp\left[i(\mathbf{k}' - \mathbf{k}) \cdot \mathbf{r}\right] \cdot (\mathbf{k}' + \mathbf{k})(a^*_{\mathbf{k}1} a_{\mathbf{k}'1} + a^*_{\mathbf{k}2} a_{\mathbf{k}'2}) \\ & + \exp\left[i(-\mathbf{k}' + \mathbf{k}) \cdot \mathbf{r}\right] \cdot (-\mathbf{k}' - \mathbf{k})(a^*_{\mathbf{k}3} a_{\mathbf{k}'3} + a^*_{\mathbf{k}4} a_{\mathbf{k}'4}) \\ & + \exp\left[i(-\mathbf{k}' - \mathbf{k}) \cdot \mathbf{r}\right] \cdot (-\mathbf{k}' + \mathbf{k})(a^*_{\mathbf{k}3} a_{\mathbf{k}'1} + a^*_{\mathbf{k}4} a_{\mathbf{k}'2}) \\ & + \exp\left[i(\mathbf{k}' + \mathbf{k}) \cdot \mathbf{r}\right] \cdot (\mathbf{k}' - \mathbf{k})(a^*_{\mathbf{k}1} a_{\mathbf{k}'3} + a^*_{\mathbf{k}2} a_{\mathbf{k}'4}) \Big\} \\ & - \frac{e^2}{mc|\Lambda|} \mathbf{A}(\mathbf{r}) \sum_{\mathbf{k}>0} (n_{\mathbf{k}1} + n_{\mathbf{k}2} + n_{\mathbf{k}3} + n_{\mathbf{k}4}). \end{aligned}$$

It follows that in order to evaluate $\mathbf{J}(\mathbf{r}, t)$ we must find the form of $a_{\mathbf{k}i}$ i $a^*_{\mathbf{k}i}$ in Heisenberg picture. To this end we must solve the Heisenberg equations (HE) of motion for $a_{\mathbf{k}i}$ i $a^*_{\mathbf{k}i}$. The HE for $a^*_{\mathbf{k}1}$ is

$$i\hbar \frac{d}{dt} a^*_{\mathbf{k}1}(t) = e^{\frac{i}{\hbar} tH} [a^*_{\mathbf{k}1}, H] e^{-\frac{i}{\hbar} tH}. \quad (2.4)$$

The commutator in equation (2.4) equals

$$[a^*_{\mathbf{k}1}, H] = -\xi_k a^*_{\mathbf{k}1} - 2\Delta P_{\mathbf{k}1}$$

where $P_{\mathbf{k}1} = a_{\mathbf{k}2} a_{\mathbf{k}3} a_{\mathbf{k}4}$. The HE for $P_{\mathbf{k}1}$ is

$$i\hbar \frac{d}{dt} P_{\mathbf{k}1}(t) = e^{\frac{i}{\hbar} tH} [P_{\mathbf{k}1}, H] e^{-\frac{i}{\hbar} tH} \quad (2.5)$$

with the commutator on the r.h.s. equal

$$[P_{\mathbf{k}1}, H] = 3\xi_k P_{\mathbf{k}1} - 2\Delta a^*_{\mathbf{k}1} \hat{O}_{\mathbf{k}1}$$

and $O_{\mathbf{k}1} := 1 - n_{\mathbf{k}2} - n_{\mathbf{k}3} - n_{\mathbf{k}4} + n_{\mathbf{k}2} n_{\mathbf{k}3} + n_{\mathbf{k}3} n_{\mathbf{k}4} + n_{\mathbf{k}2} n_{\mathbf{k}4}$. The properties of $O_{\mathbf{k}1}$ are:

1. $\hat{O}^2_{\mathbf{k}1} = \hat{O}_{\mathbf{k}1}$
2. $\hat{O}_{\mathbf{k}1} P_{\mathbf{k}1} = P_{\mathbf{k}1}$
3. $[a^*_{\mathbf{k}1}, \hat{O}_{\mathbf{k}1}] = [H, \hat{O}_{\mathbf{k}1}] = [\Lambda_\sigma, \hat{O}_{\mathbf{k}1}] = [2S, \hat{O}_{\mathbf{k}1}] = 0$

where $2S_{\mathbf{k}} = \sum_{\alpha=\pm1} \sum_{\sigma=\pm1} \sigma n_{\alpha\mathbf{k},\sigma}$, $\Lambda_{\mathbf{k}\sigma} = n_{\mathbf{k},\sigma} - n_{-\mathbf{k},-\sigma}$, $\sigma = \pm$. Commutativity of $\hat{O}_{\mathbf{k}1}$ with H and Λ_σ, $2S$ implies existence of common eigenspaces of these operators. The structure of common eigenspaces of H, Λ_σ, $2S$ is given in Table 1 in Appendix A. The matrix form of $\hat{O}_{\mathbf{k}1}$ in the basis consisting of vectors specified in the first column of Table 1 is a diagonal matrix with four elements equal to unity. The structure of this matrix shows the spectrum of $\hat{O}_{\mathbf{k}1}$: $Sp\hat{O}_{\mathbf{k}1} = \{0,1\}$. The eigen-subspace $N^1_{\mathbf{k}1}$ corresponding to the eigenvalue 1 is spanned by the vectors: $|1000\rangle$, $|0111\rangle$, $u_{\mathbf{k}}|0000\rangle + v_{\mathbf{k}}|1111\rangle$, $u_{\mathbf{k}}|1111\rangle - v_{\mathbf{k}}|0000\rangle$, whereas the eigenspace $N^1_{\mathbf{k}0}$ corresponding to the eigenvalue 0 is spanned by: $|0100\rangle$, $|0010\rangle$, $|0001\rangle$, $|1100\rangle$, $|1010\rangle$, $|1001\rangle$, $|0110\rangle$, $|0011\rangle$, $|0101\rangle$, $|1110\rangle$, $|1011\rangle$, $|1101\rangle$. $u_{\mathbf{k}}$ and $v_{\mathbf{k}}$ are the same as defined in Introduction.

Given the eigenstructure of $\hat{O}_{\mathbf{k}1}$, the problem of solving the HE can be resolved: In order to find $a^*_{\mathbf{k}1}(t)$ we must solve the following system of two coupled linear differential equations:

$$\left\{\begin{array}{ll} i\hbar\frac{d}{dt}a^*_{\mathbf{k}1}(t) & = -\xi_k a^*_{\mathbf{k}1}(t) - 2\Delta P_{\mathbf{k}1}(t) \\ i\hbar\frac{d}{dt}P_{\mathbf{k}1}(t) & = 3\xi_k P_{\mathbf{k}1}(t) - 2\Delta a^*_{\mathbf{k}1}(t)\hat{O}_{\mathbf{k}1} \end{array}\right\}. \quad (2.6)$$

Due to the vanishing of $\hat{O}_{\mathbf{k}1}$ on $N^1_{\mathbf{k}0}$ and constant value on $N^1_{\mathbf{k}1}$, as well as the equalities $\langle v_1|a^*_{\mathbf{k}1}(0)|v_0\rangle = \langle v_1|P_{\mathbf{k}1}(0)|v_0\rangle = 0$ for $v_1 \in N^1_{\mathbf{k}1}$, $v_0 \in N^1_{\mathbf{k}0}$, the equations (2.6) can be considered independently in both subspaces, viz.,

- on $N^1_{\mathbf{k}1}$ they take the form

$$\left\{\begin{array}{l} i\hbar\frac{d}{dt}a^*_{\mathbf{k}1}(t) = -\xi_k a^*_{\mathbf{k}1}(t) - 2\Delta P_{\mathbf{k}1}(t) \\ i\hbar\frac{d}{dt}P_{\mathbf{k}1}(t) = 3\xi_k P_{\mathbf{k}1}(t) - 2\Delta a^*_{\mathbf{k}1}(t) \end{array}\right\} \quad (2.7)$$

- and on $N^1_{\mathbf{k}0}$ they reduce to

$$\left\{\begin{array}{ll} i\hbar\frac{d}{dt}a^*_{\mathbf{k}1}(t) = -\xi_k a^*_{\mathbf{k}1}(t) - 2\Delta P_{\mathbf{k}1}(t) \\ i\hbar\frac{d}{dt}P_{\mathbf{k}1}(t) = 3\xi_k P_{\mathbf{k}1}(t) \end{array}\right\}. \quad (2.8)$$

The system of equations (2.8) can be solved by substituting $P_{\mathbf{k}1}$ from the first equation to the second. The resulting differential linear equation for $a^*_{\mathbf{k}1}(t)$ is

$$\frac{d^2}{dt^2}a^*_{\mathbf{k}1}(t) + 2\frac{i}{\hbar}\xi_{\mathbf{k}}\frac{d}{dt}a^*_{\mathbf{k}1}(t) + \hbar^{-2}(3\xi^2_{\mathbf{k}} + 4\Delta^2)a^*_{\mathbf{k}1}(t) = 0. \quad (2.9)$$

Under the initial conditions $P_{\mathbf{k}1}(0) = a_{\mathbf{k}2}a_{\mathbf{k}3}a_{\mathbf{k}4}$ and $a^*_{\mathbf{k}1}(0) = a^*_{\mathbf{k}1}$ one obtains the following solution:

$$\begin{aligned} a^*_{\mathbf{k}1}(t) &= (u_{\mathbf{k}}v_{\mathbf{k}}P_{\mathbf{k}1} + u^2_{\mathbf{k}}a^*_{\mathbf{k}1})e^{\frac{i}{\hbar}tE_{\mathbf{k}1}} \\ &\quad + (v^2_{\mathbf{k}}a^*_{\mathbf{k}1} - u_{\mathbf{k}}v_{\mathbf{k}}P_{\mathbf{k}1})e^{-\frac{i}{\hbar}tE_{\mathbf{k}2}} \quad (2.10) \\ a_{\mathbf{k}1}(t) &= (u_{\mathbf{k}}v_{\mathbf{k}}P^*_{\mathbf{k}1} + u^2_{\mathbf{k}}a_{\mathbf{k}1})e^{-\frac{i}{\hbar}tE_{\mathbf{k}1}} \\ &\quad + (v^2_{\mathbf{k}}a_{\mathbf{k}1} - u_{\mathbf{k}}v_{\mathbf{k}}P^*_{\mathbf{k}1})e^{\frac{i}{\hbar}tE_{\mathbf{k}2}} \end{aligned}$$

where $E_{\mathbf{k}1} = -\xi_{\mathbf{k}}+2E_{\mathbf{k}}$, $E_{\mathbf{k}2} = \xi_{\mathbf{k}}+2E_{\mathbf{k}}$, $E_{\mathbf{k}} = \sqrt{\xi^2_{\mathbf{k}} + \Delta^2}$, $u^2_{\mathbf{k}} = \frac{1}{2}(1 + \frac{\xi_k}{E_{\mathbf{k}}})$ and $v^2_{\mathbf{k}} = \frac{1}{2}(1 - \frac{\xi_k}{E_{\mathbf{k}}})$.

The second equation (2.8) in $N^1_{\mathbf{k}0}$ can be solved immediately:

$$P_{\mathbf{k}1}(t) = e^{-\frac{i}{\hbar}t3\xi_{\mathbf{k}}}P_{\mathbf{k}1}.$$

Substitution of this solution into the first equation (2.8) yields the following equation for $a^*_{\mathbf{k}1}(t)$:

$$\frac{d}{dt}a^*_{\mathbf{k}1}(t) - \frac{i}{\hbar}\xi_{\mathbf{k}}a^*_{\mathbf{k}1}(t) = \frac{i}{\hbar}2\Delta e^{-\frac{i}{\hbar}t3\xi_{\mathbf{k}}}P_{\mathbf{k}1}. \quad (2.11)$$

The solution of this equation (under the same initial conditions as before) is

$$\begin{aligned} a^*_{\mathbf{k}1}(t) &= a^*_{\mathbf{k}1}e^{\frac{i}{\hbar}t\xi_{\mathbf{k}}} + \frac{\Delta}{2\xi_{\mathbf{k}}}P_{\mathbf{k}1}(e^{\frac{i}{\hbar}t\xi_{\mathbf{k}}} - e^{-\frac{i}{\hbar}t3\xi_{\mathbf{k}}}) \quad (2.12) \\ a_{\mathbf{k}1}(t) &= a_{\mathbf{k}1}e^{-\frac{i}{\hbar}t\xi_{\mathbf{k}}} + \frac{\Delta}{2\xi_{\mathbf{k}}}P^*_{\mathbf{k}1}(e^{-\frac{i}{\hbar}t\xi_{\mathbf{k}}} - e^{\frac{i}{\hbar}t3\xi_{\mathbf{k}}}). \end{aligned}$$

The remaining operators $a^*_{\mathbf{k}i}(t)$ and $a_{\mathbf{k}i}(t)$ for $i = 2,3,4$ are given in Appendix B.

The operators $a^*_{\mathbf{k}i}(t)$ and $a_{\mathbf{k}i}(t)$ found above, fulfil the following anticommutation relations:

$$\begin{aligned} \{a^*_{\mathbf{k}i}(t), a_{\mathbf{k}'j}(t')\}_{1/0} &= \{a_{\mathbf{k}i}(t), a_{\mathbf{k}'j}(t')\}_{1/0} \\ &= \{a^*_{\mathbf{k}i}(t), a^*_{\mathbf{k}'j}(t')\}_{1/0} = 0, \text{ for } \mathbf{k}\neq\mathbf{k}', \end{aligned} \quad (2.13)$$

$$\begin{aligned} \{a^*_{\mathbf{k}i}(t), a_{\mathbf{k}i}(t')\}_1 &= u^2_{\mathbf{k}}v^2_{\mathbf{k}}(e^{\frac{i}{\hbar}tE_{\mathbf{k}1}}e^{\frac{i}{\hbar}t'E_{\mathbf{k}2}} \\ &\quad + e^{-\frac{i}{\hbar}tE_{\mathbf{k}2}}e^{-\frac{i}{\hbar}t'E_{\mathbf{k}1}}) + u^4_{\mathbf{k}}e^{\frac{i}{\hbar}(t-t')E_{\mathbf{k}1}} + v^4_{\mathbf{k}}e^{-\frac{i}{\hbar}(t-t')E_{\mathbf{k}2}} \\ &\quad + u^2_{\mathbf{k}}v^2_{\mathbf{k}}\Big[-(e^{\frac{i}{\hbar}tE_{\mathbf{k}1}}e^{\frac{i}{\hbar}t'E_{\mathbf{k}2}} + e^{-\frac{i}{\hbar}tE_{\mathbf{k}2}}e^{-\frac{i}{\hbar}t'E_{\mathbf{k}1}}) \\ &\quad + e^{\frac{i}{\hbar}(t-t')E_{\mathbf{k}1}} + e^{-\frac{i}{\hbar}(t-t')E_{\mathbf{k}2}}\Big], \end{aligned} \quad (2.14)$$

$$\{a^*_{\mathbf{k}i}(t), a_{\mathbf{k}i}(t')\}_0 = e^{\frac{i}{\hbar}(t-t')\xi_{\mathbf{k}}}, \quad (2.15)$$

where $i,j = 1,2,3,4$. and the subscript 1/0 meaning validity of the respective relation in both subspaces.

3 The Meissner effect

Our objective now is to evaluate the thermal average with respect to H of the current density using (2.2). The averages of $\hat{\mathbf{J}}_1$ and $\hat{\mathbf{J}}_2$ will be successively found with terms linear in $\mathbf{A}$ being only accounted for. Since $\hat{\mathbf{J}}_2$ is already linear in $\mathbf{A}$, it suffices to calculate the following expression:

$$\mathbf{J}_2(t,\mathbf{r}) = -\frac{e^2}{mc}\mathbf{A}(\mathbf{r})\sum_{\mathbf{k}>0}\sum_{i=1}^{4}\langle a^*_{\mathbf{k}i}(t)a_{\mathbf{k}i}(t)\rangle_{\mathbf{k}},$$

where

$$\langle A\rangle_{\mathbf{k}} := \text{ Tr } A\exp(-\beta H_{\mathbf{k}})(\text{ Tr }\exp(-\beta H_{\mathbf{k}}))^{-1}.$$

The average $\langle A\rangle_{\mathbf{k}}$ can be expressed as a sum of averages over $N^i_{\mathbf{k}1}$ and $N^i_{\mathbf{k}0}$, viz.,

$$\begin{aligned} \mathbf{J}_2(t,\mathbf{r}) = -\frac{e^2}{mc}\mathbf{A}(\mathbf{r})\sum_{\mathbf{k}>0}\sum_{i=1}^{4}(\langle a^*_{\mathbf{k}i}(t)a_{\mathbf{k}i}(t)\rangle_{N^i_{\mathbf{k}1}} \\ + \langle a^*_{\mathbf{k}i}(t)a_{\mathbf{k}i}(t)\rangle_{N^i_{\mathbf{k}0}}), \end{aligned} \quad (3.1)$$

where:

$$\langle A\rangle_{N^i_{\mathbf{k}1}} := \frac{\mathrm{Tr}_{\,N^i_{\mathbf{k}1}} A\exp(-\beta H_{\mathbf{k}})}{\mathrm{Tr}\,\exp(-\beta H_{\mathbf{k}})},$$

$$\langle A\rangle_{N^i_{\mathbf{k}0}} := \frac{\mathrm{Tr}_{\,N^i_{\mathbf{k}0}} A\exp(-\beta H_{\mathbf{k}})}{\mathrm{Tr}\,\exp(-\beta H_{\mathbf{k}})},$$

and $\mathrm{Tr}_{\,N^i_{\mathbf{k}\alpha}}$ for $\alpha = 0, 1$ denotes the trace evaluated in $N^i_{\mathbf{k}\alpha}$. Details of the calculations are given in Appendix C.

The operator $a^*_{\mathbf{k}i}(t)a_{\mathbf{k}i}(t')$ has the following form on $N^i_{\mathbf{k}1}$

$$\begin{aligned} a^*_{\mathbf{k}i}(t)a_{\mathbf{k}i}(t') &= n_{\mathbf{k}i}\Big[u^4_{\mathbf{k}}e^{\frac{i}{\hbar}(t-t')E_{\mathbf{k}1}} + v^4_{\mathbf{k}}e^{-\frac{i}{\hbar}(t-t')E_{\mathbf{k}2}} \\ &\quad + u^2_{\mathbf{k}}v^2_{\mathbf{k}}(e^{\frac{i}{\hbar}tE_{\mathbf{k}1}}e^{\frac{i}{\hbar}t'E_{\mathbf{k}2}} + e^{-\frac{i}{\hbar}tE_{\mathbf{k}2}}e^{-\frac{i}{\hbar}t'E_{\mathbf{k}1}})\Big] \\ &\quad + u_{\mathbf{k}}v_{\mathbf{k}}B^*_{\mathbf{k}}\Big[-u^2_{\mathbf{k}}e^{\frac{i}{\hbar}(t-t')E_{\mathbf{k}1}} + v^2_{\mathbf{k}}e^{-\frac{i}{\hbar}(t-t')E_{\mathbf{k}2}} \\ &\quad + u^2_{\mathbf{k}}e^{\frac{i}{\hbar}tE_{\mathbf{k}1}}e^{\frac{i}{\hbar}t'E_{\mathbf{k}2}} - v^2_{\mathbf{k}}e^{-\frac{i}{\hbar}tE_{\mathbf{k}2}}e^{-\frac{i}{\hbar}t'E_{\mathbf{k}1}}\Big] \\ &\quad + u_{\mathbf{k}}v_{\mathbf{k}}B_{\mathbf{k}}\Big[-u^2_{\mathbf{k}}e^{\frac{i}{\hbar}(t-t')E_{\mathbf{k}1}} + v^2_{\mathbf{k}}e^{-\frac{i}{\hbar}(t-t')E_{\mathbf{k}2}} \\ &\quad - v^2_{\mathbf{k}}e^{\frac{i}{\hbar}tE_{\mathbf{k}1}}e^{\frac{i}{\hbar}t'E_{\mathbf{k}2}} + u^2_{\mathbf{k}}e^{-\frac{i}{\hbar}tE_{\mathbf{k}2}}e^{-\frac{i}{\hbar}t'E_{\mathbf{k}1}}\Big] \\ &\quad + u^2_{\mathbf{k}}v^2_{\mathbf{k}}P_{\mathbf{k}i}P^*_{\mathbf{k}i}\Big[e^{\frac{i}{\hbar}(t-t')E_{\mathbf{k}1}} + e^{-\frac{i}{\hbar}(t-t')E_{\mathbf{k}2}} \\ &\quad - e^{\frac{i}{\hbar}tE_{\mathbf{k}1}}e^{\frac{i}{\hbar}t'E_{\mathbf{k}2}} - e^{-\frac{i}{\hbar}tE_{\mathbf{k}2}}e^{-\frac{i}{\hbar}t'E_{\mathbf{k}1}}\Big], \end{aligned} \tag{3.2}$$

whereas on $N^i_{\mathbf{k}0}$

$$\begin{aligned} a^*_{\mathbf{k}i}(t)a_{\mathbf{k}i}(t') &= n_{\mathbf{k}i}e^{\frac{i}{\hbar}(t-t')\xi_{\mathbf{k}}} + \frac{\Delta^2}{4\xi^2_{\mathbf{k}}}P_{\mathbf{k}i}P^*_{\mathbf{k}i} \\ &\quad \times (e^{\frac{i}{\hbar}t\xi_{\mathbf{k}}} - e^{-\frac{i}{\hbar}3t\xi_{\mathbf{k}}})(e^{-\frac{i}{\hbar}t'\xi_{\mathbf{k}}} - e^{\frac{i}{\hbar}3t'\xi_{\mathbf{k}}}) \\ &\quad - \frac{\Delta}{2\xi_{\mathbf{k}}}B^*_{\mathbf{k}}e^{\frac{i}{\hbar}t\xi_{\mathbf{k}}}(e^{-\frac{i}{\hbar}t'\xi_{\mathbf{k}}} - e^{\frac{i}{\hbar}3t'\xi_{\mathbf{k}}}) \\ &\quad - \frac{\Delta}{2\xi_{\mathbf{k}}}B_{\mathbf{k}}e^{-\frac{i}{\hbar}t'\xi_{\mathbf{k}}}(e^{\frac{i}{\hbar}t\xi_{\mathbf{k}}} - e^{-\frac{i}{\hbar}3t\xi_{\mathbf{k}}}). \end{aligned} \tag{3.3}$$

Using the expressions for the averages $\langle n_{\mathbf{k}i}\rangle_{N^i_{\mathbf{k}1}}$, $\langle B_{\mathbf{k}}\rangle_{N^i_{\mathbf{k}1}}$, $\langle P_{\mathbf{k}i}P^*_{\mathbf{k}i}\rangle_{N^i_{\mathbf{k}1}}$, $\langle n_{\mathbf{k}i}\rangle_{N^i_{\mathbf{k}0}}$, $\langle B_{\mathbf{k}}\rangle_{N^i_{\mathbf{k}0}}$ and $\langle P_{\mathbf{k}i}P^*_{\mathbf{k}i}\rangle_{N^i_{\mathbf{k}0}}$ evaluated in Appendix C, one easily finds the following averages

$$\begin{aligned} \langle a^*_{\mathbf{k}i}(t)a_{\mathbf{k}i}(t')\rangle_{N^i_{\mathbf{k}1}} &= u^2_{\mathbf{k}}\exp\left[\frac{i}{\hbar}(t-t')E_{\mathbf{k}1}\right]F_{\mathbf{k}1} \\ &\quad + v^2_{\mathbf{k}}\exp\left[-\frac{i}{\hbar}(t-t')E_{\mathbf{k}2}\right]F_{\mathbf{k}2} \end{aligned} \tag{3.4}$$

$$\langle a^*_{\mathbf{k}i}(t)a_{\mathbf{k}i}(t')\rangle_{N^i_{\mathbf{k}0}} = f_{\mathbf{k}}\exp\left[\frac{i}{\hbar}(t-t')\xi_{\mathbf{k}}\right] \tag{3.5}$$

$$\begin{aligned} \langle\{a^*_{\mathbf{k}i}(t), a_{\mathbf{k}i}(t')\}\rangle_{N^i_{\mathbf{k}1}} &= (u^2_{\mathbf{k}}\exp\left[\frac{i}{\hbar}(t-t')E_{\mathbf{k}1}\right] \\ &\quad + v^2_{\mathbf{k}}\exp\left[-\frac{i}{\hbar}(t-t')E_{\mathbf{k}2}\right])S_{\mathbf{k}} \end{aligned} \tag{3.6}$$

$$\langle\{a^*_{\mathbf{k}i}(t), a_{\mathbf{k}i}(t')\}\rangle_{N^i_{\mathbf{k}0}} = \exp\left[\frac{i}{\hbar}(t-t')\xi_{\mathbf{k}}\right]L_{\mathbf{k}} \tag{3.7}$$

where

$$F_{\mathbf{k}1} = \frac{1}{2}\frac{e^{\beta\xi_{\mathbf{k}}} + e^{-2\beta E_{\mathbf{k}}}}{3 + 4\cosh\beta\xi_{\mathbf{k}} + \cosh 2\beta E_{\mathbf{k}}},$$

$$F_{\mathbf{k}2} = \frac{1}{2}\frac{e^{\beta\xi_{\mathbf{k}}} + e^{2\beta E_{\mathbf{k}}}}{3 + 4\cosh\beta\xi_{\mathbf{k}} + \cosh 2\beta E_{\mathbf{k}}},$$

$$f_{\mathbf{k}} = \frac{3}{2}\frac{1 + e^{-\beta\xi_{\mathbf{k}}}}{3 + 4\cosh\beta\xi_{\mathbf{k}} + \cosh 2\beta E_{\mathbf{k}}},$$

$$S_{\mathbf{k}} = \frac{\cosh\beta\xi_{\mathbf{k}} + \cosh 2\beta E_{\mathbf{k}}}{3 + 4\cosh\beta\xi_{\mathbf{k}} + \cosh 2\beta E_{\mathbf{k}}}$$

and

$$L_{\mathbf{k}} = 3\frac{1 + \cosh\beta\xi_{\mathbf{k}}}{3 + 4\cosh\beta\xi_{\mathbf{k}} + \cosh 2\beta E_{\mathbf{k}}}.$$

Setting $t' = t$ in these averages and substituting them into (3.1), one obtains

$$\mathbf{J}_2(t, \mathbf{r}) = -\frac{2e^2}{mc|\Lambda|}\mathbf{A}(\mathbf{r})\sum_{\mathbf{k}}(u^2_{\mathbf{k}}F_{\mathbf{k}1} + v^2_{\mathbf{k}}F_{\mathbf{k}2} + f_{\mathbf{k}}). \tag{3.8}$$

Let us now evaluate $\mathbf{J}_1(t, \mathbf{r})$. The contribution of $\hat{\mathbf{J}}_1$ to the first term in expression (2.2) is clearly zero, as it represents the current density in equilibrium. $\mathbf{J}_1(t, \mathbf{r})$ is thus equal

$$\mathbf{J}_1(t, \mathbf{r}) = -\frac{i}{\hbar}\lim_{\substack{\tau\to\infty \\ t>\tau}}\int_0^t dt' e^{-(t-t')/\tau}\langle[\hat{\mathbf{J}}_1(\mathbf{r}, t), H'(t')]\rangle. \tag{3.9}$$

The exponential factor and limit have been introduced in order to eliminate unphysical terms which arise in the absence of natural suppression of transient currents. These currents arise at $t = 0$ and should vanish after sufficiently long time (exceeding the relaxation time τ). The limiting procedure in equation (3.9) cancels the unphysical contribution from the lower integration limit. Some details of the evaluation of the r.h.s of equation (3.9) are shown in Appendix D, where the Fourier transform $\mathbf{J}(\omega, \mathbf{q})$ is obtained as equation (D.3).

To verify whether Schafroth's criterion for the Meissner effect holds, let us now take the limits: $\tau \to \infty$, $\omega \to 0$, $\mathbf{q} \to 0$ in equation (D.3) in the same order as mentioned. Schafroth's criterion is fulfilled if equation (D.3) then reduces to

$$\mathbf{J} = -K\mathbf{a}, \text{ where } K = const > 0.$$

After passing to the limits $\tau \to \infty$, $\omega \to 0$, $\mathbf{q} \to 0$ and making use of the obvious equality

$$\begin{aligned} \lim_{y\to x}\frac{F(y)S(x) - S(y)F(x)}{y - x} &= 2S(x)\frac{dF(x)}{dx} \\ &\quad - \frac{d}{dx}F(x)S(x) = S(x)\frac{dF}{dx} - F(x)\frac{dS(x)}{dx}. \end{aligned}$$

Equation (D.3) takes the form

$$\mathbf{J}(0,0) = -8\left(\frac{e\hbar}{2m}\right)^2 \frac{1}{c|\Lambda|}\sum_{\mathbf{k}} [\mathbf{k}\cdot\mathbf{a}]\,\mathbf{k}$$
$$\times\Bigg[u_{\mathbf{k}}^4\left(S_{\mathbf{k}}\frac{\partial F_{\mathbf{k}1}}{\partial E_{\mathbf{k}1}} - F_{\mathbf{k}1}\frac{\partial S_{\mathbf{k}}}{\partial E_{\mathbf{k}1}}\right) + v_{\mathbf{k}}^4\left(F_{\mathbf{k}2}\frac{\partial S_{\mathbf{k}}}{\partial E_{\mathbf{k}2}} - S_{\mathbf{k}}\frac{\partial F_{\mathbf{k}2}}{\partial E_{\mathbf{k}2}}\right)$$
$$+ L_{\mathbf{k}}\frac{\partial f_{\mathbf{k}}}{\partial \xi_{\mathbf{k}}} - f_{\mathbf{k}}\frac{\partial L_{\mathbf{k}}}{\partial \xi_{\mathbf{k}}} + 2\frac{u_{\mathbf{k}}^2 v_{\mathbf{k}}^2}{E_{\mathbf{k}1}+E_{\mathbf{k}2}} S_{\mathbf{k}}(F_{\mathbf{k}1}-F_{\mathbf{k}2})$$
$$+ 2u_{\mathbf{k}}^2\frac{F_{\mathbf{k}1}L_{\mathbf{k}} - f_{\mathbf{k}}S_{\mathbf{k}}}{E_{\mathbf{k}1}-\xi_{\mathbf{k}}} + 2v_{\mathbf{k}}^2\frac{-F_{\mathbf{k}2}L_{\mathbf{k}} + f_{\mathbf{k}}S_{\mathbf{k}}}{E_{\mathbf{k}2}+\xi_{\mathbf{k}}}\Bigg]$$
$$- \frac{2e^2}{mc|\Lambda|}\mathbf{a}\sum_{\mathbf{k}}(u_{\mathbf{k}}^2 F_{\mathbf{k}1} + v_{\mathbf{k}}^2 F_{\mathbf{k}2} + f_{\mathbf{k}}). \quad (3.10)$$

Replacing summation over momenta in equation (3.10) by integration with respect to ξ one obtains

$$\mathbf{J}(0,0) = -\frac{e^2\sqrt{2m}}{\hbar^3 c\pi^2}\mathbf{a}\Bigg[\int_{-E_F}^{\infty}\sqrt{\xi+E_F}\,(u^2F_1+v^2F_2+f)\,d\xi$$
$$+\frac{2}{3}\int_{-E_F}^{\infty}(\xi+E_F)^{3/2}\Bigg(u^4\left(S\frac{\partial F_1}{\partial E_1} - F_1\frac{\partial S}{\partial E_1}\right)$$
$$+ v^4\left(F_2\frac{\partial S}{\partial E_2} - S\frac{\partial F_2}{\partial E_2}\right) + L\frac{\partial f}{\partial \xi} - f\frac{\partial L}{\partial \xi}$$
$$+\frac{u^2v^2}{2E}S(F_1-F_2) + u^2\frac{F_1L - fS}{E-\xi}$$
$$+ v^2\frac{-F_2L + fS}{E+\xi}\Bigg)\,d\xi\Bigg].$$

This expression can be presented in the following form

$$\mathbf{J}(0,0) = -\frac{e^2\sqrt{2m}}{\hbar^3 c\pi^2}K(0,0)\mathbf{a} \quad (3.11)$$

with

$$K(0,0) = \frac{e^2\sqrt{2m}}{\hbar^3 c\pi^2}k(0,0) \quad (3.12)$$

where

$$k(0,0) = \int_{-E_F}^{\infty}\sqrt{\xi+E_F}\,(u^2F_1+v^2F_2+f)\,d\xi$$
$$+\frac{2}{3}\int_{-E_F}^{\infty}(\xi+E_F)^{3/2}\Bigg[u^4\left(S\frac{\partial F_1}{\partial E_1} - F_1\frac{\partial S}{\partial E_1}\right)$$
$$+ v^4\left(F_2\frac{\partial S}{\partial E_2} - S\frac{\partial F_2}{\partial E_2}\right) + L\frac{\partial f}{\partial \xi} - f\frac{\partial L}{\partial \xi}$$
$$+ u^2v^2\frac{S(F_1-F_2)}{2E} + u^2\frac{F_1L - FS}{E-\xi} + v^2\frac{fS - F_2L}{E+\xi}\Bigg]\,d\xi. \quad (3.13)$$

A full proof that $k(0,0) > 0$ for any $\Delta > 0$ is extremely complex if altogether possible. Verification is, however, possible for the ground state. To this end let us note that

$$\lim_{\beta\to\infty} F_1 = 0, \quad \lim_{\beta\to\infty}\frac{\partial F_1}{\partial E_1} = 0,$$
$$\lim_{\beta\to\infty} F_2 = 1, \quad \lim_{\beta\to\infty}\frac{\partial F_2}{\partial E_2} = 0,$$
$$\lim_{\beta\to\infty} S = 1, \quad \lim_{\beta\to\infty}\frac{\partial S}{\partial E_1} = \lim_{\beta\to\infty}\frac{\partial S}{\partial E_2} = 0,$$
$$\lim_{\beta\to\infty} L = 0, \quad \lim_{\beta\to\infty}\frac{\partial L}{\partial \xi} = 0,$$

and

$$\lim_{\beta\to\infty} f = 0, \quad \lim_{\beta\to\infty}\frac{\partial f}{\partial \xi} = 0.$$

The first integral in equation (3.13) reduces to

$$\int_{-E_F}^{\infty} d\xi\sqrt{\xi+E_F}\,v^2 = \frac{2}{3}E_F^{3/2}, \quad (3.14)$$

as a result of the standard approximation $\sqrt{\xi+E_F} \approx \sqrt{E_F}$ for $\xi \in [-\delta,\delta]$. The only nonvanishing term in the second integral is the one proportional to u^2v^2, which is nonzero only on $[-\delta,\delta]$, viz.,

$$\frac{\Delta^2}{12}\int_{-\delta}^{\delta}(\xi+E_F)^{3/2}\frac{d\xi}{E^3} \approx \frac{\sqrt{E_F}\Delta^2}{12}\int_{-\delta}^{\delta}(\xi+E_F)\frac{d\xi}{E^3} = \frac{1}{6}E_F^{3/2}. \quad (3.15)$$

Combining these expressions one obtains

$$k(0,0) = \frac{1}{2}E_F^{3/2},$$

which according to (3.11) yields the following expression for the current density at $T = 0$ K:

$$\mathbf{J}(0,0) = -\frac{3}{4}\frac{ne^2}{mc}\mathbf{a}, \quad (3.16)$$

where n denotes the average number of fermions in the system. Equation (3.16) demonstrates the presence of Meissner-Ochsenfeld effect in the system with Hamiltonian H at $T = 0$.

It is worth noting that equation (3.16) differs from the corresponding equation resulting from BCS theory by factor of 3/4, which is due to the weaker character of interaction V_4 compared to the BCS one. As a consequence, the penetration depth λ of the magnetic field for H is larger than λ_{BCS} for H_{BCS}. A similar inequality $\lambda_{HTSC} > \lambda_{CL}$ holds between the penetration depths of high temperature superconductors and classical superconductors [18,19].

The question remains whether Schafroth's criterion holds for H at finite temperatures. The answer can be found by evaluating $k(0,0)$ numerically. This has been done for $g\rho_F = 0.14$, $\gamma = 0$, $\delta = 217.14 \times 10^{-4}$ eV, $E_F = 10$ eV with help of numerical results regarding parameter Δ in reference [13]. The resulting plot of $k(0,0)$

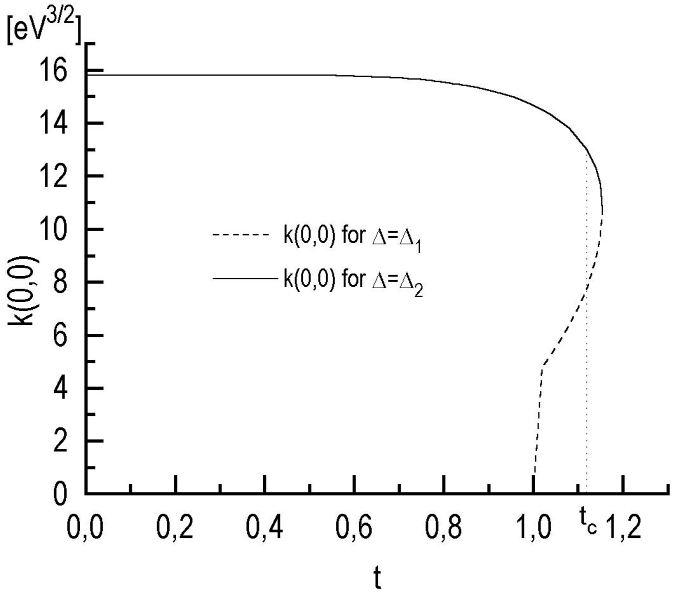

Fig. 1. The temperature dependence of $k(0,0)$ for $g\rho_F = 0.14$, $\gamma = 0$, $\delta = 217.14 \times 10^{-4}$ eV, $E_F = 10$ eV. Δ_1 and Δ_2 correspond to the nonzero solutions of the gap equation, which are plotted in [13]. $t_c = 1.12$ is the real critical temperature.

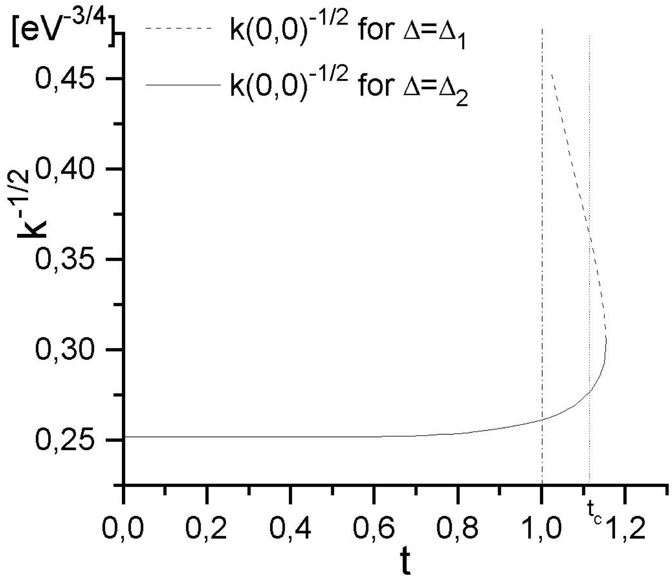

Fig. 2. The temperature dependence of $k(0,0)^{-1/2}$ for $g\rho_F = 0.14$, $\gamma = 0$, $\delta = 217.14 \times 10^{-4}$ eV, $E_F = 10$ eV. Δ_1 and Δ_2 correspond to the nonzero solutions of the gap equation, which are plotted in [13]. $t_c = 1.12$ is the real critical temperature.

versus $t = TT_1^{-1}$ (where T_1 is the temperature at which the $\Delta > 0$ solution bifurcates from the $\Delta = 0$ one)presented in Figure 1, shows that Schafroth's criterion is fulfilled at finite temperatures for the chosen values of $g\rho_F$ and δ. The same result obtains for other values of $g\rho_F$ and δ. Making use of above results one can answer the question concerning a relation between the penetration depth λ and temperature. As it is known [20] following expression occurs for the pure London superconductor

$$\frac{\lambda(T)^2}{\lambda(0)^2} = \frac{n}{n_s(T)}, \tag{3.17}$$

where n is the density of all electrons and $n_s(T)$ is the density of superconducting electrons. Equation (3.17) can be written in terms of functions $K(0,0)$, $k(0,0)$ and their values at $T = 0$. Therefore, it can be seen that

$$\lambda(t) \propto k(0,0)^{-1/2}.$$

The graph of $k(0,0)^{-1/2}$ is presented in Figure 2. $t_c = 1.12$ shown in Figures 1 and 2 is the real critical temperature in which the system undergoes the first order phase transition. A behaviour of $k(0,0)^{-1/2}$ at t_c can be interpreted as follows: when the temperature is increased the external magnetic field more and more penetrates the superconductor and at t_c rapidly fills it completely. The transition is not continuous.

4 Conclusions

The presence of a Meissner-Ochsenfeld effect has been demonstrated by Rickayzen's method [12] for a superconductor with a BCS-type 4-fermion quadruple binding potential. The effect proves to be weaker than in BCS by a factor 3/4 at $T = 0$, implying a greater penetration depth λ of external magnetic field. Moreover, the penetration depth is an increasing function of the temperature. The value of λ_{HTSC} in high temperature superconductors also exceeds the corresponding value in classical superconductors. Finally, the question is how coexistence of BCS and quadruple interactions affects the Meissner effect. At present it would be very difficult to show it for finite temperatures. Perhaps it would be possible to do it for the ground state. As suggested in Introduction such coexistence could occur where the half flux quanta appear, e.g. in $YBa_2Cu_3O_{7-\delta}$. Regarding the question of the gauge-invariance of the theory the Rickayzen's method applied does not guarantee it and further generalizations are necessary in order to incorporate invariance under gauge transformations.

This work is part of author's Ph.D. thesis written under the supervision of Prof. J. Maćkowiak.

Appendix A

$H_{\mathbf{k}}$ acts in the 16-dimensional space of states

$$(a^*_{\mathbf{k}1})^{n_1}(a^*_{\mathbf{k}2})^{n_2}(a^*_{\mathbf{k}3})^{n_3}(a^*_{\mathbf{k}4})^{n_4}|0\rangle\,,$$

where $n_i = 0, 1$, $i = 1, 2, 3, 4$. Eigenstructure of $H_{\mathbf{k}}$:

It is worth noting that the vector number 15 is the ground state vector $|G\rangle_{\mathbf{k}}$ of the system for momentum $\mathbf{k}$ because the corresponding energy has the minimal value.

Table 1.

	eigenvector	eigenvalue $H_{\mathbf{k}}$	2S	Λ_+	Λ_-
1.	$\|1000\rangle$	$\xi_{\mathbf{k}}$	1	1	0
2.	$\|0100\rangle$	$\xi_{\mathbf{k}}$	−1	0	1
3.	$\|0010\rangle$	$\xi_{\mathbf{k}}$	1	0	−1
4.	$\|0001\rangle$	$\xi_{\mathbf{k}}$	−1	−1	0
5.	$\|1010\rangle$	$2\xi_{\mathbf{k}}$	2	1	−1
6.	$\|0101\rangle$	$2\xi_{\mathbf{k}}$	−2	−1	1
7.	$\|1001\rangle$	$2\xi_{\mathbf{k}}$	0	0	0
8.	$\|0110\rangle$	$2\xi_{\mathbf{k}}$	0	0	0
9.	$\|1100\rangle$	$2\xi_{\mathbf{k}}$	0	1	1
10.	$\|0011\rangle$	$2\xi_{\mathbf{k}}$	0	−1	−1
11.	$\|1110\rangle$	$3\xi_{\mathbf{k}}$	1	1	0
12.	$\|0111\rangle$	$3\xi_{\mathbf{k}}$	−1	−1	0
13.	$\|1101\rangle$	$3\xi_{\mathbf{k}}$	−1	0	1
14.	$\|1011\rangle$	$3\xi_{\mathbf{k}}$	1	0	−1
15.	$u_{\mathbf{k}}\|0000\rangle + v_{\mathbf{k}}\|1111\rangle$	$2\xi_{\mathbf{k}} - 2E_{\mathbf{k}}$	0	0	0
16.	$u_{\mathbf{k}}\|1111\rangle - v_{\mathbf{k}}\|0000\rangle$	$2\xi_{\mathbf{k}} + 2E_{\mathbf{k}}$	0	0	0

Appendix B

The remaining operators $a^*_{\mathbf{k}i}(t)$ and $a_{\mathbf{k}i}(t)$ for $i = 2, 3, 4$ in Heisenberg picture can be found analogously. E.g. in the case of $\hat{O}_{\mathbf{k}2} := 1 - n_{\mathbf{k}1} - n_{\mathbf{k}3} - n_{\mathbf{k}4} + n_{\mathbf{k}1}n_{\mathbf{k}3} + n_{\mathbf{k}3}n_{\mathbf{k}4} + n_{\mathbf{k}1}n_{\mathbf{k}4}$ the space $N^2_{\mathbf{k}1}$ is spanned by: $|0100\rangle$, $|1011\rangle$, $u_{\mathbf{k}}|0000\rangle + v_{\mathbf{k}}|1111\rangle$, $u_{\mathbf{k}}|1111\rangle - v_{\mathbf{k}}|0000\rangle$ whereas $N^2_{\mathbf{k}0}$ by: $|1000\rangle$, $|0010\rangle$, $|0001\rangle$, $|1100\rangle$, $|1010\rangle$, $|1001\rangle$, $|0110\rangle$, $|0011\rangle$, $|0101\rangle$, $|1110\rangle$, $|0111\rangle$, $|1101\rangle$. The operators $a^*_{\mathbf{k}i}(t)$ and $a_{\mathbf{k}i}(t)$ for $i = 2, 3, 4$ are found to have the following form:

- on $N^2_{\mathbf{k}1}$

$$a^*_{\mathbf{k}2}(t) = (-u_{\mathbf{k}}v_{\mathbf{k}}P_{\mathbf{k}2} + u^2_{\mathbf{k}}a^*_{\mathbf{k}2})e^{\frac{i}{\hbar}tE_{\mathbf{k}1}} + (v^2_{\mathbf{k}}a^*_{\mathbf{k}2} + u_{\mathbf{k}}v_{\mathbf{k}}P_{\mathbf{k}2})e^{-\frac{i}{\hbar}tE_{\mathbf{k}2}}$$
$$a_{\mathbf{k}2}(t) = (-u_{\mathbf{k}}v_{\mathbf{k}}P^*_{\mathbf{k}2} + u^2_{\mathbf{k}}a_{\mathbf{k}2})e^{-\frac{i}{\hbar}tE_{\mathbf{k}1}} + (v^2_{\mathbf{k}}a_{\mathbf{k}2} + u_{\mathbf{k}}v_{\mathbf{k}}P^*_{\mathbf{k}2})e^{\frac{i}{\hbar}tE_{\mathbf{k}2}}$$

- on $N^2_{\mathbf{k}0}$

$$a^*_{\mathbf{k}2}(t) = a^*_{\mathbf{k}2}e^{\frac{i}{\hbar}t\xi_{\mathbf{k}}} - \frac{\Delta}{2\xi_{\mathbf{k}}}P_{\mathbf{k}2}(e^{\frac{i}{\hbar}t\xi_{\mathbf{k}}} - e^{-\frac{i}{\hbar}t3\xi_{\mathbf{k}}})$$
$$a_{\mathbf{k}2}(t) = a_{\mathbf{k}2}e^{-\frac{i}{\hbar}t\xi_{\mathbf{k}}} - \frac{\Delta}{2\xi_{\mathbf{k}}}P^*_{\mathbf{k}2}(e^{-\frac{i}{\hbar}t\xi_{\mathbf{k}}} - e^{\frac{i}{\hbar}t3\xi_{\mathbf{k}}})$$

with $P_{\mathbf{k}2} = a_{\mathbf{k}1}a_{\mathbf{k}3}a_{\mathbf{k}4}$. Similarly $a^*_{\mathbf{k}3}(t)$ and $a_{\mathbf{k}3}(t)$ on the spaces $N^3_{\mathbf{k}0}$ (spanned by $|1000\rangle$, $|0100\rangle$, $|0001\rangle$, $|1100\rangle$, $|1010\rangle$, $|1001\rangle$, $|0110\rangle$, $|0011\rangle$, $|0101\rangle$, $|1110\rangle$, $|0111\rangle$, $|1011\rangle$) and $N^3_{\mathbf{k}1}$ (spanned by $|0010\rangle$, $|1101\rangle$, $u_{\mathbf{k}}|0000\rangle + v_{\mathbf{k}}|1111\rangle$, $u_{\mathbf{k}}|1111\rangle - v_{\mathbf{k}}|0000\rangle$) take the form

- on $N^3_{\mathbf{k}1}$

$$a^*_{\mathbf{k}3}(t) = (u_{\mathbf{k}}v_{\mathbf{k}}P_{\mathbf{k}3} + u^2_{\mathbf{k}}a^*_{\mathbf{k}3})e^{\frac{i}{\hbar}tE_{\mathbf{k}1}} + (v^2_{\mathbf{k}}a^*_{\mathbf{k}3} - u_{\mathbf{k}}v_{\mathbf{k}}P_{\mathbf{k}3})e^{-\frac{i}{\hbar}tE_{\mathbf{k}2}}$$
$$a_{\mathbf{k}3}(t) = (u_{\mathbf{k}}v_{\mathbf{k}}P^*_{\mathbf{k}3} + u^2_{\mathbf{k}}a_{\mathbf{k}3})e^{-\frac{i}{\hbar}tE_{\mathbf{k}1}} + (v^2_{\mathbf{k}}a_{\mathbf{k}3} - u_{\mathbf{k}}v_{\mathbf{k}}P^*_{\mathbf{k}3})e^{\frac{i}{\hbar}tE_{\mathbf{k}2}}$$

- on $N^3_{\mathbf{k}0}$

$$a^*_{\mathbf{k}3}(t) = a^*_{\mathbf{k}3}e^{\frac{i}{\hbar}t\xi_{\mathbf{k}}} + \frac{\Delta}{2\xi_{\mathbf{k}}}P_{\mathbf{k}3}(e^{\frac{i}{\hbar}t\xi_{\mathbf{k}}} - e^{-\frac{i}{\hbar}t3\xi_{\mathbf{k}}})$$
$$a_{\mathbf{k}3}(t) = a_{\mathbf{k}3}e^{-\frac{i}{\hbar}t\xi_{\mathbf{k}}} + \frac{\Delta}{2\xi_{\mathbf{k}}}P^*_{\mathbf{k}3}(e^{-\frac{i}{\hbar}t\xi_{\mathbf{k}}} - e^{\frac{i}{\hbar}t3\xi_{\mathbf{k}}})$$

with $P_{\mathbf{k}3} = a_{\mathbf{k}1}a_{\mathbf{k}2}a_{\mathbf{k}4}$ and $a^*_{\mathbf{k}4}(t)$ and $a_{\mathbf{k}4}(t)$ on the spaces $N^4_{\mathbf{k}0}$ (spanned by $|1000\rangle$, $|0100\rangle$, $|0010\rangle$, $|1100\rangle$, $|1010\rangle$, $|1001\rangle$, $|0110\rangle$, $|0011\rangle$, $|0101\rangle$, $|1101\rangle$, $|0111\rangle$, $|1011\rangle$) and $N^4_{\mathbf{k}1}$ (spanned by $|0001\rangle$, $|1110\rangle$, $u_{\mathbf{k}}|0000\rangle + v_{\mathbf{k}}|1111\rangle$, $u_{\mathbf{k}}|1111\rangle - v_{\mathbf{k}}|0000\rangle$) take the form

- on $N^4_{\mathbf{k}1}$

$$a^*_{\mathbf{k}4}(t) = (-u_{\mathbf{k}}v_{\mathbf{k}}P_{\mathbf{k}4} + u^2_{\mathbf{k}}a^*_{\mathbf{k}4})e^{\frac{i}{\hbar}tE_{\mathbf{k}1}} + (v^2_{\mathbf{k}}a^*_{\mathbf{k}4} + u_{\mathbf{k}}v_{\mathbf{k}}P_{\mathbf{k}4})e^{-\frac{i}{\hbar}tE_{\mathbf{k}2}}$$
$$a_{\mathbf{k}4}(t) = (-u_{\mathbf{k}}v_{\mathbf{k}}P^*_{\mathbf{k}4} + u^2_{\mathbf{k}}a_{\mathbf{k}4})e^{-\frac{i}{\hbar}tE_{\mathbf{k}1}} + (v^2_{\mathbf{k}}a_{\mathbf{k}4} + u_{\mathbf{k}}v_{\mathbf{k}}P^*_{\mathbf{k}4})e^{\frac{i}{\hbar}tE_{\mathbf{k}2}}$$

- on $N^4_{\mathbf{k}0}$

$$a^*_{\mathbf{k}4}(t) = a^*_{\mathbf{k}4}e^{\frac{i}{\hbar}t\xi_{\mathbf{k}}} - \frac{\Delta}{2\xi_{\mathbf{k}}}P_{\mathbf{k}4}(e^{\frac{i}{\hbar}t\xi_{\mathbf{k}}} - e^{-\frac{i}{\hbar}t3\xi_{\mathbf{k}}})$$
$$a_{\mathbf{k}4}(t) = a_{\mathbf{k}4}e^{-\frac{i}{\hbar}t\xi_{\mathbf{k}}} - \frac{\Delta}{2\xi_{\mathbf{k}}}P^*_{\mathbf{k}4}(e^{-\frac{i}{\hbar}t\xi_{\mathbf{k}}} - e^{\frac{i}{\hbar}t3\xi_{\mathbf{k}}})$$

with $P_{\mathbf{k}4} = a_{\mathbf{k}1}a_{\mathbf{k}2}a_{\mathbf{k}3}$.

Appendix C

The averages occurring in equation (3.1) express in terms of $\langle n_{\mathbf{k}i}\rangle_{N^i_{\mathbf{k}1}}$, $\langle B_{\mathbf{k}}\rangle_{N^i_{\mathbf{k}1}}$, $\langle P_{\mathbf{k}i}P^*_{\mathbf{k}i}\rangle_{N^i_{\mathbf{k}1}}$, $\langle n_{\mathbf{k}i}\rangle_{N^i_{\mathbf{k}0}}$, $\langle B_{\mathbf{k}}\rangle_{N^i_{\mathbf{k}0}}$ and $\langle P_{\mathbf{k}i}P^*_{\mathbf{k}i}\rangle_{N^i_{\mathbf{k}0}}$. For $i = 1$ the averages over $N^1_{\mathbf{k}1}$ obtains from the following expectation values:

$$\langle 0001|n_{\mathbf{k}1}|1000\rangle = 1,$$
$$\langle 1110|n_{\mathbf{k}1}|0111\rangle = 0,$$
$$(v_{\mathbf{k}}\langle 1111| + u_{\mathbf{k}}\langle 0000|)n_{\mathbf{k}1}(v_{\mathbf{k}}|1111\rangle + u_{\mathbf{k}}|0000\rangle) = v^2_{\mathbf{k}},$$
$$(-v_{\mathbf{k}}\langle 0000| + u_{\mathbf{k}}\langle 1111|)n_{\mathbf{k}1}(-v_{\mathbf{k}}|0000\rangle + u_{\mathbf{k}}|1111\rangle) = u^2_{\mathbf{k}},$$
$$\langle 0001|B_{\mathbf{k}1}|1000\rangle = 0,$$
$$\langle 1110|B_{\mathbf{k}1}|0111\rangle = 0,$$
$$(v_{\mathbf{k}}\langle 1111| + u_{\mathbf{k}}\langle 0000|)B_{\mathbf{k}1}(v_{\mathbf{k}}|1111\rangle + u_{\mathbf{k}}|0000\rangle) = u_{\mathbf{k}}v_{\mathbf{k}},$$
$$(-v_{\mathbf{k}}\langle 0000| + u_{\mathbf{k}}\langle 1111|) \times B_{\mathbf{k}1}(-v_{\mathbf{k}}|0000\rangle + u_{\mathbf{k}}|1111\rangle) = -u_{\mathbf{k}}v_{\mathbf{k}},$$
$$\langle 0001|P_{\mathbf{k}1}P^*_{\mathbf{k}1}|1000\rangle = 1,$$
$$\langle 1110|P_{\mathbf{k}1}P^*_{\mathbf{k}1}|0111\rangle = 0,$$
$$(v_{\mathbf{k}}\langle 1111| + u_{\mathbf{k}}\langle 0000|)P_{\mathbf{k}1}P^*_{\mathbf{k}1}(v_{\mathbf{k}}|1111\rangle + u_{\mathbf{k}}|0000\rangle) = u^2_{\mathbf{k}},$$

$$(-v_{\mathbf{k}}\langle 0000| + u_{\mathbf{k}}\langle 1111|) \times P_{\mathbf{k}1}P^*_{\mathbf{k}1}(-v_{\mathbf{k}}|0000\rangle + u_{\mathbf{k}}|1111\rangle) = v^2_{\mathbf{k}},$$

whereas those over $N^1_{\mathbf{k}0}$ express in terms of:

$$\langle 0010|n_{\mathbf{k}1}|0100\rangle = \langle 0100|n_{\mathbf{k}1}|0010\rangle = \langle 1000|n_{\mathbf{k}1}|0001\rangle = 0,$$
$$\langle 0011|n_{\mathbf{k}1}|1100\rangle = \langle 0101|n_{\mathbf{k}1}|1010\rangle = \langle 1001|n_{\mathbf{k}1}|1001\rangle = 1,$$
$$\langle 0110|n_{\mathbf{k}1}|0110\rangle = \langle 1100|n_{\mathbf{k}1}|0011\rangle = \langle 1010|n_{\mathbf{k}1}|0101\rangle = 0,$$
$$\langle 0111|n_{\mathbf{k}1}|1110\rangle = \langle 1101|n_{\mathbf{k}1}|1011\rangle = \langle 1011|n_{\mathbf{k}1}|1101\rangle = 1,$$

$$\langle 0010|B_{\mathbf{k}1}|0100\rangle = \langle 0100|B_{\mathbf{k}1}|0010\rangle = \langle 1000|B_{\mathbf{k}1}|0001\rangle = \langle 0011|B_{\mathbf{k}1}|1100\rangle = \langle 0101|B_{\mathbf{k}1}|1010\rangle = \langle 1001|B_{\mathbf{k}1}|1001\rangle = \langle 0110|B_{\mathbf{k}1}|0110\rangle = \langle 1100|B_{\mathbf{k}1}|0011\rangle = \langle 1010|B_{\mathbf{k}1}|0101\rangle = \langle 0111|B_{\mathbf{k}1}|1110\rangle = \langle 1101|B_{\mathbf{k}1}|1011\rangle = \langle 1011|B_{\mathbf{k}1}|1101\rangle = 0,$$

$$\langle 0010|P_{\mathbf{k}1}P^*_{\mathbf{k}1}|0100\rangle = \langle 0100|P_{\mathbf{k}1}P^*_{\mathbf{k}1}|0010\rangle = \langle 1000|P_{\mathbf{k}1}P^*_{\mathbf{k}1}|0001\rangle = \langle 0011|P_{\mathbf{k}1}P^*_{\mathbf{k}1}|1100\rangle = \langle 0101|P_{\mathbf{k}1}P^*_{\mathbf{k}1}|1010\rangle = \langle 1001|P_{\mathbf{k}1}P^*_{\mathbf{k}1}|1001\rangle = \langle 0110|P_{\mathbf{k}1}P^*_{\mathbf{k}1}|0110\rangle = \langle 1100|P_{\mathbf{k}1}P^*_{\mathbf{k}1}|0011\rangle = \langle 1010|P_{\mathbf{k}1}P^*_{\mathbf{k}1}|0101\rangle = \langle 0111|P_{\mathbf{k}1}P^*_{\mathbf{k}1}|1110\rangle = \langle 1101|P_{\mathbf{k}1}P^*_{\mathbf{k}1}|1011\rangle = \langle 1011|P_{\mathbf{k}1}P^*_{\mathbf{k}1}|1101\rangle = 0.$$

Using these results, one obtains

$$\langle n_{\mathbf{k}1}\rangle_{N^1_{\mathbf{k}1}} = \frac{e^{-\beta\xi_{\mathbf{k}}} + v^2_{\mathbf{k}}e^{-2\beta(\xi_{\mathbf{k}}-E_{\mathbf{k}})} + u^2_{\mathbf{k}}e^{-2\beta(\xi_{\mathbf{k}}+E_{\mathbf{k}})}}{4e^{-\beta\xi_{\mathbf{k}}}+6e^{-2\beta\xi_{\mathbf{k}}}+4e^{-3\beta\xi_{\mathbf{k}}}+e^{-2\beta(\xi_{\mathbf{k}}-E_{\mathbf{k}})}+e^{-2\beta(\xi_{\mathbf{k}}+E_{\mathbf{k}})}},$$

$$\langle B_{\mathbf{k}}\rangle_{N^1_{\mathbf{k}1}} = \langle B^*_{\mathbf{k}}\rangle_{N^1_{\mathbf{k}1}} = \frac{u_{\mathbf{k}}v_{\mathbf{k}}e^{-2\beta(\xi_{\mathbf{k}}-E_{\mathbf{k}})} - u_{\mathbf{k}}v_{\mathbf{k}}e^{-2\beta(\xi_{\mathbf{k}}+E_{\mathbf{k}})}}{4e^{-\beta\xi_{\mathbf{k}}}+6e^{-2\beta\xi_{\mathbf{k}}}+4e^{-3\beta\xi_{\mathbf{k}}}+e^{-2\beta(\xi_{\mathbf{k}}-E_{\mathbf{k}})}+e^{-2\beta(\xi_{\mathbf{k}}+E_{\mathbf{k}})}},$$

$$\langle P_{\mathbf{k}1}P^*_{\mathbf{k}1}\rangle_{N^1_{\mathbf{k}1}} = \frac{e^{-\beta\xi_{\mathbf{k}}} + u^2_{\mathbf{k}}e^{-2\beta(\xi_{\mathbf{k}}-E_{\mathbf{k}})} + v^2_{\mathbf{k}}e^{-2\beta(\xi_{\mathbf{k}}+E_{\mathbf{k}})}}{4e^{-\beta\xi_{\mathbf{k}}}+6e^{-2\beta\xi_{\mathbf{k}}}+4e^{-3\beta\xi_{\mathbf{k}}}+e^{-2\beta(\xi_{\mathbf{k}}-E_{\mathbf{k}})}+e^{-2\beta(\xi_{\mathbf{k}}+E_{\mathbf{k}})}},$$

$$\langle n_{\mathbf{k}1}\rangle_{N^1_{\mathbf{k}0}} = \frac{3e^{-2\beta\xi_{\mathbf{k}}} + 3e^{-3\beta\xi_{\mathbf{k}}}}{4e^{-\beta\xi_{\mathbf{k}}}+6e^{-2\beta\xi_{\mathbf{k}}}+4e^{-3\beta\xi_{\mathbf{k}}}+e^{-2\beta(\xi_{\mathbf{k}}-E_{\mathbf{k}})}+e^{-2\beta(\xi_{\mathbf{k}}+E_{\mathbf{k}})}},$$

$$\langle B_{\mathbf{k}}\rangle_{N^1_{\mathbf{k}0}} = \langle B^*_{\mathbf{k}}\rangle_{N^1_{\mathbf{k}0}} = 0,$$
$$\langle P_{\mathbf{k}1}P^*_{\mathbf{k}1}\rangle_{N^1_{\mathbf{k}0}} = 0.$$

The averages $\langle\rangle_{N^i_{\mathbf{k}1}}$, $\langle\rangle_{N^i_{\mathbf{k}0}}$ for $i = 2, 3, 4$ can be found analogously.

Appendix D

The r.h.s of equation (3.9) can be evaluated using the identity

$$[a^*_{\mathbf{k}i}(t)a_{\mathbf{k}'j}(t), a^*_{\mathbf{k}'j}(t')a_{\mathbf{k}i}(t')] = a^*_{\mathbf{k}i}(t)a_{\mathbf{k}i}(t')\{a^*_{\mathbf{k}'j}(t'), a_{\mathbf{k}'j}(t)\} - a^*_{\mathbf{k}'j}(t')a_{\mathbf{k}'j}(t)\{a^*_{\mathbf{k}i}(t), a_{\mathbf{k}i}(t')\},$$

which follows from the relationships (2.14) for $\mathbf{k} \neq \mathbf{k}'$. The fourfold sum over momenta which arises in equation (3.9) reduces to a double sum. Equation (3.9) then takes the form

See equation (D.1) next page.

The contribution to this sum over $\mathbf{k}' = \mathbf{k}$ can be neglected, as it reduces to a double integral over a set of zero measure. Each product of averages in the r.h.s. of equation (D.1) is next replaced by a sum of products of averages over appropriate subspaces. Furthermore, making use of the averages (3.4)–(3.7), and Fourier transformation $\mathbf{a}(\mathbf{k}' - \mathbf{k}, t) = \int d\omega e^{-i\omega t}\mathbf{a}(\mathbf{k}' - \mathbf{k}, \omega)$ and performing integration over t', one obtains

See equation (D.2) next page.

We replaced the sum $\sum_{\substack{\mathbf{k}>0\\ \mathbf{k}'>0}}$ by unrestricted sum $\sum_{\mathbf{k}\mathbf{k}'}$ and neglected terms proportional to $e^{-t/\tau}$ $(t > \tau)$. Let us now

$$\mathbf{J}_1(t,\mathbf{r}) = \frac{i}{\hbar}\left(\frac{e\hbar}{2m}\right)^2 \frac{1}{c|\Lambda|} \lim_{\substack{\tau\to\infty \\ t>\tau}} \sum_{\substack{\mathbf{k}>0 \\ \mathbf{k}'>0}} \int dt' e^{-(t-t')/\tau} \Big\{ \left[e^{i(\mathbf{k}'-\mathbf{k})\cdot\mathbf{r}} (\mathbf{k}'+\mathbf{k})\cdot\mathbf{a}(\mathbf{k}'-\mathbf{k},t)\right](\mathbf{k}'+\mathbf{k})$$

$$\times \big(\langle a^*_{\mathbf{k}1}(t)a_{\mathbf{k}1}(t')\rangle_{\mathbf{k}} \left\langle\left\{a^*_{\mathbf{k}'1}(t'),a_{\mathbf{k}'1}(t)\right\}\right\rangle_{\mathbf{k}'} - \left\langle a^*_{\mathbf{k}'1}(t')a_{\mathbf{k}'1}(t)\right\rangle_{\mathbf{k}'} \left\langle\left\{a^*_{\mathbf{k}1}(t),a_{\mathbf{k}1}(t')\right\}\right\rangle_{\mathbf{k}}$$

$$+\langle a^*_{\mathbf{k}2}(t)a_{\mathbf{k}2}(t')\rangle_{\mathbf{k}} \left\langle\left\{a^*_{\mathbf{k}'2}(t'),a_{\mathbf{k}'2}(t)\right\}\right\rangle_{\mathbf{k}'} - \left\langle a^*_{\mathbf{k}'2}(t')a_{\mathbf{k}'2}(t)\right\rangle_{\mathbf{k}'} \left\langle\left\{a^*_{\mathbf{k}2}(t),a_{\mathbf{k}2}(t')\right\}\right\rangle_{\mathbf{k}}\big)$$

$$+\left[e^{i(-\mathbf{k}'+\mathbf{k})\cdot\mathbf{r}} (-\mathbf{k}'-\mathbf{k})\cdot\mathbf{a}(-\mathbf{k}'+\mathbf{k},t)\right](-\mathbf{k}'-\mathbf{k})$$

$$\times \big(\left\langle a^*_{\mathbf{k}3}(t)a_{\mathbf{k}3}(t')\right\rangle_{\mathbf{k}} \left\langle\left\{a^*_{\mathbf{k}'3}(t'),a_{\mathbf{k}'3}(t)\right\}\right\rangle_{\mathbf{k}'} - \left\langle a^*_{\mathbf{k}'3}(t')a_{\mathbf{k}'3}(t)\right\rangle_{\mathbf{k}'} \left\langle\left\{a^*_{\mathbf{k}3}(t),a_{\mathbf{k}3}(t')\right\}\right\rangle_{\mathbf{k}}$$

$$+\langle a^*_{\mathbf{k}4}(t)a_{\mathbf{k}4}(t')\rangle_{\mathbf{k}} \left\langle\left\{a^*_{\mathbf{k}'4}(t'),a_{\mathbf{k}'4}(t)\right\}\right\rangle_{\mathbf{k}'} - \left\langle a^*_{\mathbf{k}'4}(t')a_{\mathbf{k}'4}(t)\right\rangle_{\mathbf{k}'} \left\langle\left\{a^*_{\mathbf{k}4}(t),a_{\mathbf{k}4}(t')\right\}\right\rangle_{\mathbf{k}}\big)$$

$$+\left[e^{i(-\mathbf{k}'-\mathbf{k})\cdot\mathbf{r}} (-\mathbf{k}'+\mathbf{k})\cdot\mathbf{a}(-\mathbf{k}'-\mathbf{k},t)\right](-\mathbf{k}'+\mathbf{k})$$

$$\times \big(\left\langle a^*_{\mathbf{k}1}(t)a_{\mathbf{k}1}(t')\right\rangle_{\mathbf{k}} \left\langle\left\{a^*_{\mathbf{k}'3}(t'),a_{\mathbf{k}'3}(t)\right\}\right\rangle_{\mathbf{k}'} - \left\langle a^*_{\mathbf{k}'3}(t')a_{\mathbf{k}'3}(t)\right\rangle_{\mathbf{k}'} \left\langle\left\{a^*_{\mathbf{k}1}(t),a_{\mathbf{k}1}(t')\right\}\right\rangle_{\mathbf{k}}$$

$$+\left\langle a^*_{\mathbf{k}2}(t)a_{\mathbf{k}2}(t')\right\rangle_{\mathbf{k}} \left\langle\left\{a^*_{\mathbf{k}'4}(t'),a_{\mathbf{k}'4}(t)\right\}\right\rangle_{\mathbf{k}'} - \left\langle a^*_{\mathbf{k}'4}(t')a_{\mathbf{k}'4}(t)\right\rangle_{\mathbf{k}'} \left\langle\left\{a^*_{\mathbf{k}2}(t),a_{\mathbf{k}2}(t')\right\}\right\rangle_{\mathbf{k}}\big)$$

$$+\left[e^{i(\mathbf{k}'+\mathbf{k})\cdot\mathbf{r}} (\mathbf{k}'-\mathbf{k})\cdot\mathbf{a}(\mathbf{k}'+\mathbf{k},t)\right](\mathbf{k}'-\mathbf{k})$$

$$\times \big(\left\langle a^*_{\mathbf{k}3}(t)a_{\mathbf{k}3}(t')\right\rangle_{\mathbf{k}} \left\langle\left\{a^*_{\mathbf{k}'1}(t'),a_{\mathbf{k}'1}(t)\right\}\right\rangle_{\mathbf{k}'} - \left\langle a^*_{\mathbf{k}'1}(t')a_{\mathbf{k}'1}(t)\right\rangle_{\mathbf{k}'} \left\langle\left\{a^*_{\mathbf{k}3}(t),a_{\mathbf{k}3}(t')\right\}\right\rangle_{\mathbf{k}}$$

$$+\left\langle a^*_{\mathbf{k}2}(t)a_{\mathbf{k}2}(t')\right\rangle_{\mathbf{k}} \left\langle\left\{a^*_{\mathbf{k}'4}(t'),a_{\mathbf{k}'4}(t)\right\}\right\rangle_{\mathbf{k}'} - \left\langle a^*_{\mathbf{k}'4}(t')a_{\mathbf{k}'4}(t)\right\rangle_{\mathbf{k}'} \left\langle\left\{a^*_{\mathbf{k}2}(t),a_{\mathbf{k}2}(t')\right\}\right\rangle_{\mathbf{k}}\Big\}. \tag{D.1}$$

$$\mathbf{J}_1(t,\mathbf{r}) = -2\left(\frac{e\hbar}{2m}\right)^2 \frac{1}{c|\Lambda|} \lim_{\tau\to\infty} \sum_{\mathbf{k}\mathbf{k}'} e^{i(\mathbf{k}'-\mathbf{k})\cdot\mathbf{r}} \int d\omega e^{-i\omega t} \left[(\mathbf{k}'+\mathbf{k})\cdot\mathbf{a}(\mathbf{k}'-\mathbf{k},\omega)\right](\mathbf{k}'+\mathbf{k})$$

$$\times \Bigg[u^2_{\mathbf{k}}u^2_{\mathbf{k}'} \frac{F_{\mathbf{k}1}S_{\mathbf{k}'} - F_{\mathbf{k}'1}S_{\mathbf{k}}}{i\hbar/\tau + \hbar\omega - (E_{\mathbf{k}'1} - E_{\mathbf{k}1})} + v^2_{\mathbf{k}}v^2_{\mathbf{k}'} \frac{F_{\mathbf{k}2}S_{\mathbf{k}'} - F_{\mathbf{k}'2}S_{\mathbf{k}}}{i\hbar/\tau + \hbar\omega + (E_{\mathbf{k}'2} - E_{\mathbf{k}2})}$$

$$+ \frac{f_{\mathbf{k}}L_{\mathbf{k}'} - f_{\mathbf{k}'}L_{\mathbf{k}}}{i\hbar/\tau + \hbar\omega - (\xi_{\mathbf{k}'} - \xi_{\mathbf{k}})} + u^2_{\mathbf{k}}v^2_{\mathbf{k}'} \frac{F_{\mathbf{k}1}S_{\mathbf{k}'}}{i\hbar/\tau + \hbar\omega + (E_{\mathbf{k}1} + E_{\mathbf{k}'2})}$$

$$+ v^2_{\mathbf{k}}u^2_{\mathbf{k}'} \frac{F_{\mathbf{k}2}S_{\mathbf{k}'}}{i\hbar/\tau + \hbar\omega - (E_{\mathbf{k}2} + E_{\mathbf{k}'1})} + u^2_{\mathbf{k}} \frac{F_{\mathbf{k}1}L_{\mathbf{k}'}}{i\hbar/\tau + \hbar\omega + (E_{\mathbf{k}1} - \xi_{\mathbf{k}'})}$$

$$- u^2_{\mathbf{k}'}v^2_{\mathbf{k}} \frac{F_{\mathbf{k}'1}S_{\mathbf{k}}}{i\hbar/\tau + \hbar\omega - (E_{\mathbf{k}'1} + E_{\mathbf{k}2})} - u^2_{\mathbf{k}}v^2_{\mathbf{k}'} \frac{F_{\mathbf{k}'2}S_{\mathbf{k}}}{i\hbar/\tau + \hbar\omega + (E_{\mathbf{k}'2} + E_{\mathbf{k}1})}$$

$$+ v^2_{\mathbf{k}} \frac{F_{\mathbf{k}2}L_{\mathbf{k}'}}{i\hbar/\tau + \hbar\omega - (E_{\mathbf{k}2} + \xi_{\mathbf{k}'})} + u^2_{\mathbf{k}'} \frac{f_{\mathbf{k}}S_{\mathbf{k}'}}{i\hbar/\tau + \hbar\omega + (\xi_{\mathbf{k}} - E_{\mathbf{k}'1})}$$

$$+ v^2_{\mathbf{k}'} \frac{f_{\mathbf{k}}S_{\mathbf{k}}}{i\hbar/\tau + \hbar\omega - (\xi_{\mathbf{k}} + E_{\mathbf{k}'2})} - u^2_{\mathbf{k}'} \frac{F_{\mathbf{k}'1}L_{\mathbf{k}}}{i\hbar/\tau + \hbar\omega + (\xi_{\mathbf{k}} - E_{\mathbf{k}'1})}$$

$$- v^2_{\mathbf{k}'} \frac{F_{\mathbf{k}'2}L_{\mathbf{k}}}{i\hbar/\tau + \hbar\omega + (\xi_{\mathbf{k}} + E_{\mathbf{k}'2})} - u^2_{\mathbf{k}} \frac{f_{\mathbf{k}'}S_{\mathbf{k}}}{i\hbar/\tau + \hbar\omega + (E_{\mathbf{k}1} - \xi_{\mathbf{k}'})} - v^2_{\mathbf{k}} \frac{f_{\mathbf{k}'}S_{\mathbf{k}}}{i\hbar/\tau + \hbar\omega - (E_{\mathbf{k}2} + \xi_{\mathbf{k}'})}\Bigg]. \tag{D.2}$$

$$\mathbf{J}\left(\omega,\mathbf{q}\right) = -2\left(\frac{e\hbar}{2m}\right)^2 \frac{1}{c|\Lambda|} \lim_{\tau\to\infty} \sum_{\mathbf{k}} \left[(\mathbf{q}+2\mathbf{k})\cdot\mathbf{a}(\mathbf{q},\omega)\right](\mathbf{q}+2\mathbf{k})$$
$$\times\Bigg[u_{\mathbf{k}}^2 u_{\mathbf{k+q}}^2 \frac{F_{\mathbf{k}1}S_{\mathbf{k+q}} - F_{\mathbf{k+q}1}S_{\mathbf{k}}}{i\hbar/\tau + \hbar\omega - (E_{\mathbf{k+q}1} - E_{\mathbf{k}1})} + v_{\mathbf{k}}^2 v_{\mathbf{k+q}}^2 \frac{F_{\mathbf{k}2}S_{\mathbf{k+q}} - F_{\mathbf{k+q}2}S_{\mathbf{k}}}{i\hbar/\tau + \hbar\omega + (E_{\mathbf{k+q}2} - E_{\mathbf{k}2})}$$
$$+ u_{\mathbf{k}}^2 v_{\mathbf{k+q}}^2 \frac{F_{\mathbf{k}1}S_{\mathbf{k+q}}}{i\hbar/\tau + \hbar\omega + (E_{\mathbf{k}1} + E_{\mathbf{k+q}2})} + v_{\mathbf{k}}^2 u_{\mathbf{k+q}}^2 \frac{F_{\mathbf{k}2}S_{\mathbf{k+q}}}{i\hbar/\tau + \hbar\omega - (E_{\mathbf{k}2} + E_{\mathbf{k+q}1})}$$
$$- u_{\mathbf{k+q}}^2 v_{\mathbf{k}}^2 \frac{F_{\mathbf{k+q}1}S_{\mathbf{k}}}{i\hbar/\tau + \hbar\omega - (E_{\mathbf{k+q}1} + E_{\mathbf{k}2})} - u_{\mathbf{k}}^2 v_{\mathbf{k+q}}^2 \frac{F_{\mathbf{k+q}2}S_{\mathbf{k}}}{i\hbar/\tau + \hbar\omega + (E_{\mathbf{k+q}2} + E_{\mathbf{k}1})}$$
$$+ \frac{f_{\mathbf{k}}L_{\mathbf{k+q}} - f_{\mathbf{k+q}}L_{\mathbf{k}}}{i\hbar/\tau + \hbar\omega - (\xi_{\mathbf{k+q}} - \xi_{\mathbf{k}})} + u_{\mathbf{k}}^2 \frac{F_{\mathbf{k}1}L_{\mathbf{k+q}}}{i\hbar/\tau + \hbar\omega + (E_{\mathbf{k}1} - \xi_{\mathbf{k+q}})}$$
$$+ v_{\mathbf{k}}^2 \frac{F_{\mathbf{k}2}L_{\mathbf{k+q}}}{i\hbar/\tau + \hbar\omega - (E_{\mathbf{k}2} + \xi_{\mathbf{k+q}})} + u_{\mathbf{k+q}}^2 \frac{f_{\mathbf{k}}S_{\mathbf{k+q}}}{i\hbar/\tau + \hbar\omega + (\xi_{\mathbf{k}} - E_{\mathbf{k+q}1})}$$
$$+ v_{\mathbf{k+q}}^2 \frac{f_{\mathbf{k}}S_{\mathbf{k+q}}}{i\hbar/\tau + \hbar\omega - (\xi_{\mathbf{k}} + E_{\mathbf{k+q}2})} - u_{\mathbf{k+q}}^2 \frac{F_{\mathbf{k+q}1}L_{\mathbf{k}}}{i\hbar/\tau + \hbar\omega + (\xi_{\mathbf{k}} - E_{\mathbf{k+q}1})}$$
$$- v_{\mathbf{k+q}}^2 \frac{F_{\mathbf{k+q}2}L_{\mathbf{k}}}{i\hbar/\tau + \hbar\omega + (\xi_{\mathbf{k}} + E_{\mathbf{k+q}2})} - u_{\mathbf{k}}^2 \frac{f_{\mathbf{k+q}}S_{\mathbf{k}}}{i\hbar/\tau + \hbar\omega + (E_{\mathbf{k}1} - \xi_{\mathbf{k+q}})}$$
$$- v_{\mathbf{k}}^2 \frac{f_{\mathbf{k+q}}S_{\mathbf{k}}}{i\hbar/\tau + \hbar\omega - (E_{\mathbf{k}2} + \xi_{\mathbf{k+q}})}\Bigg] - \frac{2e^2}{mc}\mathbf{a}(\mathbf{q},\omega)\sum_{\mathbf{k}}(u_{\mathbf{k}}^2 F_{\mathbf{k}1} + v_{\mathbf{k}}^2 F_{\mathbf{k}2} + f_{\mathbf{k}}). \qquad \text{(D.3)}$$

introduce the Fourier transform $\mathbf{J}(\mathbf{q},\omega)$ of the current density using (2.3)

See equation (D.3) above.

References

1. J. Maćkowiak, P. Tarasewicz, Mol. Phys. Rep. **15/16**, 61 (1996)
2. J. Spałek, W. Wójcik, Phys. Rev. B **37**, 1532 (1988)
3. W. Wójcik, J. Spałek, `cond-mat/9406095`
4. J. Czerwonko, Physica C **235-240**, 2337 (1994)
5. J. Czerwonko, Mol. Phys. Rep. **12**, 79 (1995)
6. T. Schneider, H. Keller, Int. J. Mod. Phys. B **8**, 487 (1993)
7. Yu.M. Bunkov, A.S. Chen, D.J. Cousins, H. Godfrin, Phys. Rev. Lett. **85**, 3456 (2000)
8. G.E. Volovik, *Exotic Properties of Superfluid ^3He* (World Scientific, Singapore, 1992)
9. C.W. Schneider, G. Hammerl, G. Logvenov, T. Kopp, J.R. Kirtley, P.J. Hirschfeld, J. Mannhart, `cond-mat/0403595`
10. J. Maćkowiak, P. Tarasewicz, Mol. Phys. Rep. **20**, 145 (1997)
11. J. Maćkowiak, P. Tarasewicz, Acta Phys. Polonica A **93**, 659 (1998)
12. J. Maćkowiak, P. Tarasewicz, Physica C **331**, 25 (2000)
13. P. Tarasewicz, J. Maćkowiak, Physica C **329**, 130 (2000)
14. P. Tarasewicz, accepted in J. Superconductivity
15. K.A. Müller, Nature **377**, 133 (1995)
16. J. Schrieffer, *Theory of Superconductivity* (W.A. Benjamin, Inc., New York, 1964)
17. G. Rickayzen, *Theory of Superconductivity* (Interscience, New York, London, 1965)
18. M. Cyrot, D. Pavuna, *Introduction to Superconductivity and High-T_c Materials* (World Scientific Publishing Co. Pte. Ltd., 1992)
19. Ch.P. Poole, H.A. Farach, R.J. Creswick, *Superconductivity* (Academic Press, Inc. New York, London, 1995)
20. A.L. Fetter, J.D. Walecka, *Quantum Theory of Many-Particle Systems* (McGraw-Hill, Inc. New York, 1971)